General Method for the Kinematical Modeling of the Hexapodous Robots

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Abstract - A method for the kinematic study of the spatial mechanisms, based on a matrix formalism, easy to adjust to the computer's modeling is presented in the paper. This method is part of the authors' efforts to develop a general algorithm for a complete study of the mechanisms with deformable links. The method is applied on the kinematic chain of a hexapodous robot whose structure and movement had been inspired from the study of a hexapodous insect. The theoretic research has been materialized through an experimental model which complies faithfully with the structure and the movement of the biomechanism.

I. INTRODUCTION

Some of the most important achievements of the authors in the field of the dynamic modeling of the spatial mechanisms, especially applied on the walking robots, are presented in the papers [1], [2], [3].

These methods have a complex character, based on mathematical models which allow the full description of the kinematic elements' movement, but especially of the geometry and the kinematic and dynamic behaviour of the kinematic pairs, in order to obtain an integrated system, useful in computer aided design, of the plane and spatial mechanisms. But, unfortunately these methods are available only for mobile mechanical systems with rigid elements.

We have in view to conclude a general method for the kinematic study of the plane and spatial mechanisms, to allow the subsequent consideration of the elements' deformability, both for the kinematic and dynamic problems.

The kinematic elements' shape is defined by introducing some simple configurated transfer matrix.

II. THE GENERAL KINEMATIC MODEL

It's considered a kinematic linkage made by n rigid solids, connected through n-1 kinematic pairs (fig.1).

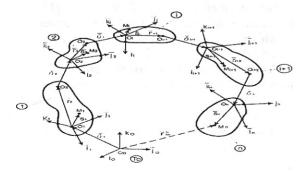


Fig. 1. The kinematic linkage

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We make the next notations:

 $-T_i(\overline{i_i},\overline{j_i},\overline{k_i})$ - the reference frame attached to the element i. with unit vectors base $\overline{W}_i(i_i, \overline{j}_i, \overline{k}_i)$; $i = \overline{1, n}$.

- $T_0(\tilde{i}_i, \overline{j}_i, \overline{k}_i)$ - the global reference frame with unit vectors base $\overline{W}_0(\overline{i_0},\overline{j_0},\overline{k_0});$

- δ_i - the relative translation vector between the elements i-1 and i, with respect to existed trihedral, if there is a prismatic pair between elements i-1 and i; (i = 1, n).

- r_i - the position vector with respect to the reference frame Ti, of O'I point, from which begins the relative translation, (i = 1, n);

 $-\overrightarrow{S_i}$ - the position vector of M i, in proportion to T_i ,

A. Positions

The position vector of M_n point with respect to the global reference frame is given by the relationship:

$$\overrightarrow{r}_{M_n}^{T_o} = \overrightarrow{O_o M_n} = \sum_{i=1}^n (\overrightarrow{r}_i + \overrightarrow{\delta}_i) + \overrightarrow{S_n}$$
 (1)

where:
$$\vec{r}_{i} = \left\{ r_{i}^{x}, r_{i}^{y}, r_{i}^{z} \right\}_{i-1} = \left\{ r_{i} \right\}^{T} \left\{ \overline{W}_{i-1} \right\}$$
 (2)

$$\vec{\delta}_i = \left\{ \delta_i^x, \delta_i^y, \delta_i^z \right\}_{i-1} = \left\{ \delta_i \right\}^T \left\{ \overrightarrow{W}_{i-1} \right\}$$
 (3)

$$\overrightarrow{S_n} = \left\{ S_n^x, S_n^y, S_n^z \right\}_{i-1} = \left\{ S_n \right\}^T \left\{ \overrightarrow{W}_{i-1} \right\}$$
 (4)

We introduce the coordinate transformation matrix from a reference frame to another. The relations (2), (3) and (4) become:

$$\left\{\overrightarrow{W}_{i-1}\right\} = \left[A_{oi-1}\right] \left\{\overrightarrow{W}_{o}\right\} \tag{5}$$

$$\overrightarrow{r_i} = \left\{r_i\right\}^T \left\{ \overrightarrow{W}_{i-1} \right\} = \left\{r_i\right\}^T \left[A_{oi-1}\right] \left\{ \overrightarrow{W}_o \right\} \tag{6}$$

$$\overrightarrow{\delta_i} = \{\delta_i\}^T \left\{ \overrightarrow{W}_{i-1} \right\} = \left\{\delta_i\right\}^T \left[A_{oi-1}\right] \left\{ \overrightarrow{W}_o \right\} \quad (7)$$

$$\overrightarrow{S_n} = \{S_n\}^T \{\overrightarrow{W}_{i-1}\} = \{S_n\}^T [A_{oi-1}] \{\overrightarrow{W}_{i-1}\}$$
 (8) Introducing the relations (6), (7) and (8) in relation (1) we

$$\vec{r}_{M_n}^{T_o} = \overline{O_o M_n} = \left(\sum_{i=1}^n \left(\left\{ \left[\left\{ r_i \right\}^T + \left\{ \delta_i \right\}^T \right] \left[A_{oi-1} \right] + \right) + \left\{ \overline{W}_0 \right\} (9) \right)$$

B. Velocities

We get the velocities deriving with respect to time the Considering relationship (9). the coordinates transformation matrix is being orthogonal we can write the relation:

$$[A_{oi}] [A_{oi}]^T = [I]$$
 (10) Deriving with respect to time the relation (10) we obtain:

$$[\dot{A}_{oi}]A_{oi}]^T + [A_{oi}]\dot{A}_{oi}]^T = 0$$
 (11)

$$\begin{bmatrix} \dot{A}_{oi} & A_{oi}^T \end{bmatrix}^T = \begin{bmatrix} A_{oi} & \dot{A}_{oi} \end{bmatrix}^T = -\begin{bmatrix} \dot{A}_{oi} & A_{oi} \end{bmatrix}^T \tag{12}$$

 $\left[\dot{A}_{oi} \left[A_{oi}^T \right]^T = \left[A_{oi} \right] \left[\dot{A}_{oi} \right]^T = - \left[\dot{A}_{oi} \right] \left[A_{oi} \right]^T$ (12) We observe that the term $\left[\dot{A}_{oi} \right] A_{oi}^T \right]$ is an anti-symmetrical matrix:

$$[\widetilde{\omega}_{oi}] = [A_{oi}]A_{oi}]^T$$
 (13)
Multiplying the relation (13) with $[A_{oi}]$ it's obtaining:

$$\left[\widetilde{\omega}_{o,i}\right]A_{oi} = \left[\dot{A}_{oi}\right]$$
 Deriving with respect to time the relation (9) we obtain:

$$\bar{r}_{Mo}^{To} = \sum_{i=1}^{n} \left(\begin{cases} \left\{ r_{i} \right\}^{T} \left[\tilde{\omega}_{o,i-1} \right] \left[A_{oi-1} \right] + \left\{ \dot{\delta}_{i} \right\}^{T} \left[A_{oi-1} \right] + \\ + \left\{ \delta_{i} \right\}^{T} \left[\tilde{\omega}_{o,i-1} \right] \left[A_{o-i} \right] \\ + \left\{ S_{n} \right\}^{T} \left[\tilde{\omega}_{o,n} \right] \left[A_{on} \right] \end{cases} \right) + \left\{ \bar{W}_{on} \right\} (15)$$

$$\vec{\tilde{r}}_{Mo}^{To} = \sum_{i=1}^{n} \left(\left\{ r_{i} \right\}^{T} \left[\tilde{\omega}_{o,i-1} \right] + \left\{ \dot{\delta}_{i} \right\}^{T} + \left\{ A_{ol-1} \right] + \left\{ \delta_{i} \right\}^{T} \left[\tilde{\omega}_{o,i-1} \right] + \left\{ S_{n} \right\}^{T} \left[\tilde{\omega}_{o,n} \right] \left[A_{on} \right] \right\} \right) \left(16 \right)$$

We have the anti-symmetrical matrix:

$$\left[\widetilde{\omega}_{o,p} \right] = \begin{bmatrix} 0 & \omega_{0p}^z & -\omega_{0p}^y \\ -\omega_{0p}^z & 0 & \omega_{0p}^x \\ \omega_{0p}^y & -\omega_{0p}^x & 0 \end{bmatrix}$$
 (17)

where:

$$\vec{\omega}_{o,p} = \omega_{o,p}^{x} \vec{i} + \omega_{o,p}^{y} \vec{j} + \omega_{o,p}^{z} \vec{k}$$
 (18)

For every vector $\overline{\delta}_i$, \overline{r}_i and \overline{S}_i , $(i=\overline{1,n})$ we can attach an anti-symmetrical matrix, as in the relation (17). The terms used in the relation (16) can be also written as follows:

$$\{r_i\}^T \left[\widetilde{\omega}_{o,i-1}\right] = \left\{\omega_{o,i-1}\right\}^T \left[\widetilde{r}_i\right] \tag{19}$$

$$\{\delta_i\}^T [\widetilde{\omega}_{o,i-1}] = \{\omega_{o,i-1}\}^T [\widetilde{\delta}_i]$$
 (20)

$$\{\mathbf{S}_{\mathbf{n}}\}^{\mathsf{T}} \left[\widetilde{\boldsymbol{\omega}}_{0,i-1}\right] = \{\boldsymbol{\omega}_{0,i-1}\}^{\mathsf{T}} \left[\widetilde{\mathbf{S}}_{\mathbf{n}}\right]$$

$$\{\boldsymbol{\omega}_{0p}\} = \{\boldsymbol{\omega}_{p}^{\mathsf{X}} \boldsymbol{\omega}_{p}^{\mathsf{Y}} \boldsymbol{\omega}_{p}^{\mathsf{Z}}\}^{\mathsf{T}}$$

$$(21)$$

In this case we can write the relation (16):

$$\vec{V}_{Mn}^{To} = \left(\sum_{i=1}^{n} \left\{\omega_{0i-1}\right\}^{T} \begin{bmatrix} \tilde{r}_{i} \end{bmatrix} [A_{0i-1}] + \left\{\overline{W}_{0}\right\} + \left\{\sum_{i=1}^{n} \left\{\delta_{i}^{i}\right\}^{T} [A_{0i-1}]\right\} + \left\{\sum_{i=1}^{n} \left\{\omega_{0i-1}\right\}^{T} \begin{bmatrix} \tilde{\delta}_{i} \end{bmatrix} [A_{0i-1}] + \left\{\omega_{0n}\right\}^{T} \begin{bmatrix} \tilde{S}_{n} \end{bmatrix} [A_{0n}] \right\} \left\{\overline{W}_{0}\right\}$$
(22)

C. Accelerations

Deriving with respect to time the relation (22) we obtain:

$$\vec{a}_{M_{n}}^{T_{0}} = \sum_{i=1}^{n} \left\{ r_{i} \right\}^{T} \left[\widetilde{\omega}_{0i-1} \right] \left[A_{0i-1} \right] \left\{ \overline{W}_{0} \right\} + \\
+ \sum_{i=1}^{n} \left\{ r_{i} \right\}^{T} \left[\widetilde{\omega}_{0i-1} \right] \left[\widetilde{\omega}_{0i-1} \right] \left[A_{0i-1} \right] \left\{ \overline{W}_{0} \right\} + \sum_{i=1}^{n} \left\{ \widecheck{\delta}_{i} \right\}^{T} \left[A_{0i-1} \right] \left\{ \overline{W}_{0} \right\} + \\
+ \sum_{i=1}^{n} \left\{ \widecheck{\delta}_{i} \right\}^{T} \left[\widetilde{\omega}_{0i-1} \right] \left[A_{0i-1} \right] \left\{ \overline{W}_{0} \right\} + \left[\sum_{i=1}^{n} \left\{ \delta_{i} \right\}^{T} \left[\widecheck{\omega}_{0i-1} \right] \left[A_{0i-1} \right] \right] \left\{ \overline{W}_{0} \right\} + \\
+ \left(\left[\widetilde{\omega}_{0i-1} \right] \left[\widetilde{\omega}_{0i-1} \right] \left[A_{0i-1} \right] \left[A_{0i-1} \right] \right) \left\{ \overline{W}_{0} \right\} + \\
+ \left(\left\{ S_{n} \right\}^{T} \left[\widecheck{\omega}_{0i-1} \right] \left[A_{0n} \right] + \left\{ S_{n} \right\}^{T} \left[\widetilde{\omega}_{0n} \right] \left[A_{0n} \right] \left[A_{0n} \right] \right\} \left\{ \overline{W}_{0} \right\} \right. \tag{23}$$

III. THE KINEMATIC MODELING OF A HEXAPODOUS ROBOT

The method presented above is illustrated by applying it on a kinematic chain belonging to the structure of a hexapodous robot. The inspiration source for the kinematic modeling of this robot has been the movement of a hexapodous insect. Analysing the movement of this insect we've got a data base which allowed the identification of the variation laws for the generalized coordinates. This problem is detailed in the paper [3].

The general kinematic model which allows the displacement in the most general conditions is presented in



Fig. 2. The general kinematic model

The structural synthesis of this model [1] led to the feasibleness of manufacturing an experimental model which faithfully respects the structure and the step principles of the hexapodous insect (fig.3).

The experimental model has the capability to assure the locomotion on plane ground, straight line, being driven by a single motor.

To modify the mobility degree and to multiply the locomotion possibilities we can easy introduce some other motors because of the construction's flexibility.

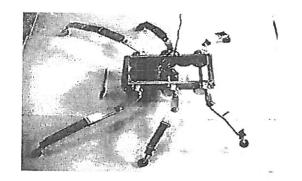


Fig. 3. The experimental model

In the paper it is proposed a structural model full compatible with the experimental one, for which, in a first stage, we determine the kinematic parameters.

IV. THE KINEMATIC ANALYSIS OF THE LEFT FORE-LEG

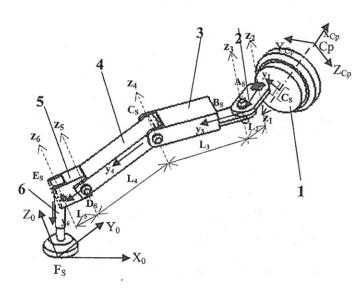


Fig. 4. The model of the left fore-leg

The connection's order of the kinematic elements is:

$$Cp-1-2-3-4-5-6$$

 $Cp-1'-1-2'-2-3'-3-4'-4-5'-5-6'-6$

The transformation matrices of the coordinates are:

$$\begin{bmatrix} C_{pl'} \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix} = \begin{bmatrix} 0 & 0 & -l \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} (24)$$

$$[A_{l'I}] = \begin{bmatrix} \cos q_I & \sin q_I & 0 \\ -\sin q_I & \cos q_I & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 (25)

$$[C_{12'}] = \begin{bmatrix} \cos\frac{\pi}{2} & 0 & \sin\frac{\pi}{2} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{2} & 0 & \cos\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$
(26)

$$[A_{2'2}] = \begin{bmatrix} \cos q_2 & \sin q_2 & 0 \\ -\sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 (27)

$$\begin{bmatrix} C_{23'} \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$
 (28)

$$[A_{3'3}] = \begin{bmatrix} \cos q_3 & \sin q_3 & 0 \\ -\sin q_3 & \cos q_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 (29)

$$[C_{34'}] = \begin{bmatrix} \cos\frac{\pi}{2} & 0 & \sin\frac{\pi}{2} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{2} & 0 & \cos\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$
(30)

$$[A_{4'4}] = \begin{bmatrix} \cos q_4 & \sin q_4 & 0 \\ -\sin q_4 & \cos q_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 (31)

$$[A_{5'5}] = \begin{bmatrix} \cos q_5 & \sin q_5 & 0 \\ -\sin q_5 & \cos q_5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 (33)

$$[C_{56'}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{34}$$

$$[A_{6'6}] = \begin{bmatrix} \cos q_6 & \sin q_6 & 0 \\ -\sin q_6 & \cos q_6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 (35)

The base changes are defined as follows:

$$\left\{ \overline{W_{I'}} \right\} = \left[C_{pI'} \right] \mathbb{E} \left\{ \overline{W_{C_p}} \right\} \qquad \qquad \left\{ \overline{W_I} \right\} = \left[A_{I'I} \right] \mathbb{E} \left\{ \overline{W_{I'}} \right\}$$

$$\left\{ \overline{W_{2'}} \right\} = \left[C_{12'} \right] \left\{ \overline{W_1} \right\} \qquad \left\{ \overline{W_2} \right\} = \left[A_{2'2} \right] \left\{ \overline{W_{2'}} \right\}$$

$$\left\{\overrightarrow{W_{3'}}\right\} = \left[C_{23'}\right] \left[\left\{\overrightarrow{W_2}\right\}\right]$$

$$\left\{ \overrightarrow{W_{3}} \right\} = \left[A_{3'3} \right] \left[\left\{ \overrightarrow{W_{3'}} \right\} \right]$$

$$\left\{\overline{W_{4}}\right\} = \left[A_{4'4}\right] \left[C_{34'}\right] \left[A_{Cp3}\right] \left\{\overline{W_{C_{p}}}\right\} = \left[A_{Cp4}\right] \left\{\overline{W_{C_{p}}}\right\}$$

$$\left\{ \overline{W_{4'}} \right\} = \left[C_{34'} \right] \left[\left\{ \overline{W_3} \right\} \right]$$

$$\left\{ \overrightarrow{W_{4}}\right\} =\left\lceil A_{4'4}\right\rceil \left[\left\langle \overrightarrow{W_{4'}}\right\rangle$$

$$\left\{\overline{W_{5}}\right\} = \left[A_{5'5}\right] \left[C_{45'}\right] \left[A_{Cp4}\right] \left\{\overline{W_{C_{p}}}\right\} = \left[A_{Cp5}\right] \left\{\overline{W_{C_{p}}}\right\}$$

$$\left\{ \overrightarrow{W_{5'}} \right\} = \left[C_{45'} \right] \left[\left\{ \overrightarrow{W_4} \right\} \right]$$

$$\left\{\overrightarrow{W_5}\right\} = \left[A_{5'5}\right] \left[\left(\overrightarrow{W_{5'}}\right)\right]$$

$$\left\{\overline{W_{6}}\right\} = \left[A_{6'6}\right] \left[C_{56'}\right] \left[A_{Cp5}\right] \left\{\overline{W_{C_{p}}}\right\} = \left[A_{Cp6}\right] \left\{\overline{W_{C_{p}}}\right\}$$

$\left\{ \overrightarrow{W_{6'}} \right\} = \left[C_{56'} \right] \left[\left\{ \overrightarrow{W_5} \right\} \right]$

$$\left\{ \overline{W_{6}} \right\} = \left[A_{6'6} \right] \left[\left\{ \overline{W_{6'}} \right\} \right]$$

A. Positions

 $\overrightarrow{r_{C_I}} = \overrightarrow{C_p C_I} = \begin{bmatrix} 0, 0, L_{C_I} \end{bmatrix}^T \square \left\{ \overrightarrow{W_{I'}} \right\}$ (36)

The position of the point F_S in the reference system solidary to the pronot is:

$$\overrightarrow{r_{C_I}} = \left\{ r_{C_I} \right\}^T \mathbb{I} \left\{ \overline{W_{I'}} \right\} = \left\{ r_{C_I} \right\}^T \mathbb{I} \left[C_{pI'} \right] \left\{ \overline{W_{C_p}} \right\}$$
(37)

$$\overline{r_{C_p}^{F_S}} = \overline{C_p F_S} = \left\{ r_{C_I} \right\}^T \square \left[C_{pI'} \right] \square \left[\overline{W_{C_p}} \right] + \left\{ r_2 \right\}^T \square \left[A_{CpI} \right] \square \left[\overline{W_{C_p}} \right] + \left\{ r_3 \right\}^T \square \left[A_{Cp2} \right] \square \left[\overline{W_{C_p}} \right] + \left\{ r_4 \right\}^T \square \left[A_{Cp3} \right] \square \left[\overline{W_{C_p}} \right] + \left\{ r_4 \right\}^T \square \left[A_{Cp3} \right] \square \left[\overline{W_{C_p}} \right] + \left\{ r_4 \right\}^T \square \left[A_{Cp3} \right] \square \left[\overline{W_{C_p}} \right] + \left\{ r_4 \right\}^T \square \left[\overline{W_{C_p}} \right] + \left\{ \overline{W_{C_p}} \right] + \left\{ \overline{W_{C_p}} \right\} + \left\{ \overline{W_{C_p}} \right\} + \left\{ \overline{W_{C_p}} \right\} - \left\{ \overline{W_{C_p}} \right\} + \left\{ \overline{W_{C_p}} \right\} - \left\{ \overline{W_{$$

$$\overrightarrow{r_2} = \overrightarrow{C_1 A_S} = \begin{bmatrix} 0, R_I, L_A \end{bmatrix}^T \square \left\{ \overrightarrow{W_I} \right\}$$

$$+\{r_{s}\}^{T} \square A_{Cp4} \square \{\overline{W_{C_{p}}}\} + \{r_{6}\}^{T} \square A_{Cp5} \square \{\overline{W_{C_{p}}}\} +$$

$$\overline{r_2} = \{r_2\}^T \square \{\overline{W_I}\} = \{r_2\}^T \square \{A_{CpI}\} \square \{\overline{W_{C_p}}\}$$
(39)

$$+\left\{S_{6}\right\}^{T}\left[A_{Cp6}\right]\left\{\overline{W_{C_{p}}}\right\}$$

$$\overrightarrow{r_3} = \overrightarrow{A_S B_S} = [0, L_2, 0]^T \square \{\overrightarrow{W_2}\}$$

(38)

(44)

$$\overline{r_3} = \{r_3\}^T \lfloor \overline{W_2} \} = \{r_3\}^T \lfloor \overline{W_C} \rfloor \langle \overline{W_C} \rangle$$

$$(41)$$

$$\overline{V_{C_p}^{F_S}} = \left\{r_2\right\}^T \left[\left[\mathcal{D}_{CpI}\right] \left[A_{CpI}\right] \left[\overline{W_{C_p}}\right] + \left\{r_3\right\}^T \left[\left[\mathcal{D}_{Cp2}\right] \left[A_{Cp2}\right] \left[\overline{W_{C_p}}\right] + \left\{r_3\right\}^T \left[\left[\mathcal{D}_{Cp2}\right] \left[\mathcal{D}_{Cp2}\right] \left[\overline{W_{C_p}}\right] + \left\{r_3\right\}^T \left[\left[\mathcal{D}_{Cp2}\right] \left[\mathcal{D}_{Cp2}\right] \left[\mathcal{D}_{C$$

(53)

$$\overline{r_4} = \overline{B_S C_S} = [0, L_3, 0]^T \mathbb{I}[\overline{W_3}]$$
(42)

$$+ \big\{ r_4 \big\}^T \mathbb{I} \left[\varpi_{Cp3} \right] \mathbb{I} \left[A_{Cp3} \right] \Big\{ \overline{W_{C_p}} \Big\} + \big\{ r_5 \big\}^T \mathbb{I} \left[\varpi_{Cp4} \right] \mathbb{I} \left[A_{Cp4} \right] \Big\{ \overline{W_{C_p}} \Big\} +$$

$$\overrightarrow{r_4} = \left\{r_4\right\}^T \left[\left\{\overline{W_3}\right\} = \left\{r_4\right\}^T \left[\left[A_{Cp3}\right]\right] \left[\left\{\overline{W_{C_p}}\right\}\right]$$
(43)

$$+\left\{r_{6}\right\}^{T}\left[\left[\varnothing_{Cp5}\right]\left[A_{Cp5}\right]\left\{\overline{W_{C_{p}}}\right\}+\left\{S_{6}\right\}^{T}\left[\left[\varnothing_{Cp6}\right]\left[A_{Cp6}\right]\left\{\overline{W_{C_{p}}}\right\}\right]\right]$$
(54)

$$\overrightarrow{r_S} = \overline{C_S D_S} = [0, L_4, 0]^T \square \{ \overline{W_4} \}$$

The antisymmetrical matrices $\left[A_{Cpi}\right]$, $i=\overline{2,6}$ have the following form:

$$\overrightarrow{r_5} = \left\{r_5\right\}^T \left[\left\{\overrightarrow{W_4}\right\} = \left\{r_5\right\}^T \left[\left[A_{Cp4}\right]\right] \left\{\left\{\overrightarrow{W_{C_p}}\right\}\right]$$
(45)

$$\begin{bmatrix} \omega_{Cpi} \end{bmatrix} = \begin{bmatrix} 0 & \omega_{Cpi}^z & -\omega_{Cpi}^y \\ -\omega_{Cpi}^z & 0 & \omega_{Cpi}^x \\ \omega_{Cpi}^y & -\omega_{Cpi}^x & 0 \end{bmatrix}$$
 (55)

$$\overrightarrow{r_6} = \overrightarrow{D_S E_S} = [0, L_5, 0]^T \left[\overline{W_5} \right]$$
(46)

$$\overline{r_6} = \left\{ r_6 \right\}^T \left[\left\{ \overline{W_5} \right\} = \left\{ r_6 \right\}^T \left[\left[A_{CP5} \right] \right] \left[\left\{ \overline{W_{C_p}} \right\} \right]$$
(47)

$$\overrightarrow{\omega_{Cpi}} = \omega_{Cpi}^x \vec{i} + \omega_{Cpi}^y \vec{j} + \omega_{Cpi}^z \vec{k} \qquad i = \overline{2,6}$$

$$\overline{S_6} = \overline{E_S F} = [0, L_6, 0]^T \mathbb{E}[\overline{W_6}]$$
(48)

$$\begin{bmatrix} \Gamma_{i} \\ r_{i} \end{bmatrix} = \begin{bmatrix} 0 & r_{i}^{z} & -r_{i}^{y} \\ -r_{i}^{z} & 0 & r_{i}^{x} \\ r_{i}^{y} & -r_{i}^{x} & 0 \end{bmatrix}; \quad i = \overline{2,6}$$
 (56)

$$\overline{S_6} = \left\{ S_6 \right\}^T \left[\left\{ \overline{W_6} \right\} = \left\{ S_6 \right\}^T \left[\left[A_{Cp6} \right] \right] \left[\overline{W_{C_p}} \right]$$
(49)

The following relationships can also be used:

$$\left\{\overline{W}_{I}\right\} = \left[A_{I'I}\right] \left[C_{pI'}\right] \left\{\overline{W}_{C_{p}}\right\} = \left[A_{CpI}\right] \left\{\overline{W}_{C_{p}}\right\} \tag{50}$$

$$\{r_i\}^T \square \left[\omega_{Cp,i-1} \right] = \left\{ \omega_{Cp,i-1} \right\}^T \square \left[r_i \right] \tag{57}$$

$$\left\{\overline{W_{2}}\right\} = \left[A_{2'2}\right] \left[C_{12'}\right] \left[A_{Cpl}\right] \left\{\overline{W_{C_{p}}}\right\} = \left[A_{Cp2}\right] \left\{\overline{W_{C_{p}}}\right\} (51)$$

In such conditions the velocity of the point F_S is:

$$\left\{\overline{W_3}\right\} = \left[A_{3'3}\right] \left[C_{23'}\right] \left[A_{C_{p2}}\right] \left\{\overline{W_{C_p}}\right\} = \left[A_{C_{p3}}\right] \left\{\overline{W_{C_p}}\right\} (52)$$

$$\begin{split} & \overline{V_{C_p}^{F_S}} = \left\{ \omega_{Cp1} \right\}^T \mathbb{Q} \left[\widehat{Y_2} \right] \left[A_{Cp1} \right] \left\{ \overline{W_{C_p}} \right\} + \\ & + \left\{ \omega_{Cp2} \right\}^T \mathbb{Q} \left[\widehat{Y_3} \right] \left[A_{Cp2} \right] \left\{ \overline{W_{C_p}} \right\} + \\ & + \left\{ \omega_{Cp3} \right\}^T \mathbb{Q} \left[\widehat{Y_4} \right] \left[A_{Cp3} \right] \left\{ \overline{W_{C_p}} \right\} + \\ & + \left\{ \omega_{Cp4} \right\}^T \mathbb{Q} \left[\widehat{Y_5} \right] \left[A_{Cp4} \right] \left\{ \overline{W_{C_p}} \right\} + \\ & + \left\{ \omega_{Cp5} \right\}^T \mathbb{Q} \left[\widehat{Y_6} \right] \left[A_{Cp5} \right] \left\{ \overline{W_{C_p}} \right\} + \\ & + \left\{ \omega_{Cp6} \right\}^T \mathbb{Q} \left[\widehat{S_6} \right] \left[A_{Cp6} \right] \left\{ \overline{W_{C_p}} \right\} \end{split}$$

(58)

(59)

$$\begin{bmatrix} \vec{S}_6 \end{bmatrix} = \begin{bmatrix} 0 & S_6^z & -S_6^y \\ -S_6^z & 0 & S_6^x \\ S_6^y & -S_6^x & 0 \end{bmatrix}$$

$$\left\{\omega_{Cpi}\right\}^T = \left\{\omega_{Cpi}^x, \omega_{Cpi}^y, \omega_{Cpi}^z\right\}^T; \ i = \overline{2,6}$$

C. Accelerations

$$\overline{a_{C_p}^{F_S}} = \{r_2\}^T \left[\overrightarrow{\omega_{C_p 1}} \right] \left[A_{C_p 1} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{r_2\}^T \left[\overrightarrow{\omega_{C_p 1}} \right] \left[\overrightarrow{\omega_{C_p 1}} \right] \left[A_{C_p 1} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{r_3\}^T \left[\overrightarrow{\omega_{C_p 2}} \right] \left[A_{C_p 2} \right] \left[\overrightarrow{W_{C_p}} \right] + \\
+ \{r_3\}^T \left[\overrightarrow{\omega_{C_p 2}} \right] \left[\overrightarrow{\omega_{C_p 2}} \right] \left[A_{C_p 2} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{r_4\}^T \left[\overrightarrow{\omega_{C_p 3}} \right] \left[\overrightarrow{\omega_{C_p 3}} \right] \left[A_{C_p 3} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{r_4\}^T \left[\overrightarrow{\omega_{C_p 3}} \right] \left[\overrightarrow{\omega_{C_p 3}} \right] \left[A_{C_p 3} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{r_5\}^T \left[\overrightarrow{\omega_{C_p 4}} \right] \left[\overrightarrow{\omega_{C_p 4}} \right] \left[A_{C_p 4} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{r_6\}^T \left[\overrightarrow{\omega_{C_p 5}} \right] \left[\overrightarrow{\omega_{C_p 5}} \right] \left[A_{C_p 5} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{S_4\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[A_{C_p 6} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{A_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \left\{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \left\{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \left\{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \left\{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \left\{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \left\{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p}} \right\} + \\
+ \left\{S_6\}^T \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left[\overrightarrow{\omega_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p 6}} \right] \left\{ \overrightarrow{W_{C_p 6}} \right\} \right\}$$

IV. THE MATHEMATICAL MODELS PROCESSING

Based on the above mathematical models it was elaborated a calculus program which allows the processing of the movement laws for each kinematic element, or characteristic point, with respect to the reference system attached to the insect's body or to the world reference system. The program is useful for getting the trajectory for every point belonging to the body with respect to the world reference system, or the path for every leg with respect to the reference system attached to the body. The movement laws of the coordinates in the kinematic pairs, established by analysing the insect's movement and the mechanism geometry, have been considered as input data.

Further on, there are presented some of the variation laws of the kinematic parameters (positions, velocities and accelerations) for the left fore-leg.

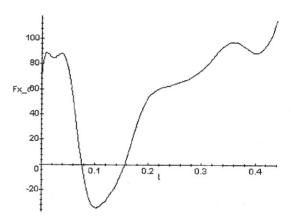


Fig. 5. The variation of the position r_x of the point F given the body

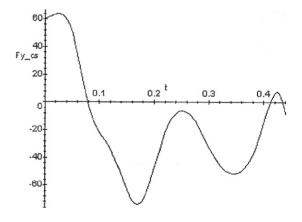


Fig.6. The variation of the position r_y of the point F given the body

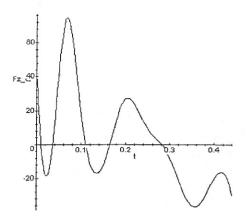


Fig.7. The variation of the position r_z of the point F given the body

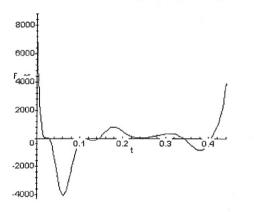


Fig. 8. The variation of the velocity v_x of the point F given the body

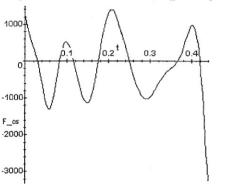


Fig. 9. The variation of the velocity v_y of the point F given the body

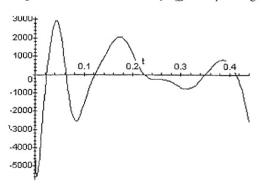


Fig. 10. The variation of the velocity v z of the point F given the body

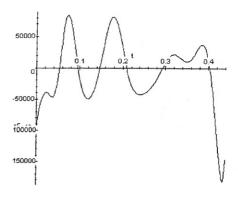


Fig. 11. The variation of the acceleration a_y of the point F given the body

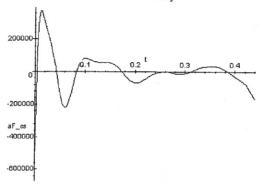


Fig.12. The variation of the acceleration a_z of the point F given the body

V. CONCLUSIONS

It is noticed the present method in the paper being very flexible, it can be used in different structural combinations. We've had in view to increase the kinematic possibilities of the experimental model, counting on the data base get through the morphological and kinematic analysis of the insect's biomechanism. Simultaneously it was made the kinematic modeling and the simulation of the robot's movement in the 3D space using the package programs Visual Nastran (fig.4). The method is useful for a further modelling of the contact with the support surface when the last element of every leg is considered to be elastically.

VI. REFERENCES

- [1] Dumitru,N., Considerations about the structural-kinematic modeling of the hexapodous robots, pag.121-126,SYROM'97.
- [2] Dumitru N., CherciuM., Contributions upon the structural-kinematic modeling of a six legged walking robot, Proceedings of international Conference on Mechanical Transmissions and Mechanisms (MTM'97),pag. 891-895,1-4 iulie 1997, Tianjin, China.
- [3] Dumitru, N., Spatial mechanisms. Computerized kinematic and dynamic modeling, "Universitaria Publishing House, Craiova, 1999.
- [4] Fichter, E., Fichter, B., Walking machinees and Mechanisms, Milano, 1995.
- [5] Gorinevsky, D.M. Sneider, A. Yu, Force Control of Legged Vehicles overrigid and soft Surfaces, The International Journal of Robotics Research, Vol. 9. No. 2. april, 1990.