A Low Cost Solution for the Navigation Problem of Wheeled Mobile Robots in Intelligent Space

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Abstract – **The paper deals with the development of low cost PI-fuzzy controllers (PI-FCs) used as tracking controllers for nonholonomic mobile robots operating in the Intelligent Space. A simplified dynamic model of the wheeled mobile robots with two degrees of freedom is first proposed, operating in a cascaded control system structure with two control loops. The PI-FCs are considered as a special class of two degree-of-freedom controllers. There is proposed also a development method dedicated to the PI-FCs starting from the basic linear PI controllers by employing the Extended Symmetrical Optimum method. There are derived sensitivity models with respect to the parametric variations of the controlled plant. Simulation results validate the proposed fuzzy controllers.**

I. INTRODUCTION

Although there have been achieved great progresses, mobile robots which are capable of performing various and complex tasks in an autonomous and intelligent way, have not yet been able to conquer widespread applications. Therefore, it is important to develop high performance controllers to cope with the three basic navigation problems [1], tracking control (tracking a reference trajectory), path following and point stabilization. The tracking control problem can be further divided in the local and global tracking problems [2]. These control problems belong to the general class of controlling nonsmooth or nonholonomic systems [3], [4]. The control of nonholonomic mobile robots has received much research interest during the past years (see, for example, the overview presented in [3]) due to the implications of the nonholonomic constraints on the admissible control inputs for this class of systems.

The majority of controllers developed for nonholonomic mobile robots is based on kinematic or dynamic models [5], [6]. But, the dynamic models do not exploit the dynamics of the actuators, of the measuring devices and of the control equipment as part of the control system (abbreviated as CS). This comes to the first aim of the paper, the proposal of a simplified dynamic model that can characterize well the wheeled mobile robots with two degrees of freedom.

The current approaches to solving tracking control problems include general nonlinear techniques [7] with the backstepping as part of them [2], [4], the sliding mode approach [5], linear model or passivity based approaches [8], [9], the control Lyapunov function approach [10]. Some efforts regarding the application of sliding mode control have been reported [11], [12]. Since the development of the controllers based on these approaches is rather complex, it is necessary to simplify the controller development and further implementation. This will result in the second aim of this paper, to offer a development method for the PI-fuzzy controllers (PI-FCs) as a special class of the general class of two degree-of-freedom (2-DOF) controllers, used as tracking controllers. The development method is based on applying the Extended Symmetrical Optimum (ESO) method [13] to the basic linear PI controllers and by adding nonlinear features to the strictly speaking fuzzy blocks to improve the CS performance.

The CS proposed here to solve the tracking control problem contains two control loops for controlling the forward velocity and the angle between the heading direction and the x-axis. The reference inputs for these two control loops are obtained by first applying the artificial potential field method used in obstacle avoidance [14] for generating the reference trajectory of the robot. Then, there are performed simple computations that employ the tracking errors for the x- and y- axes and the maximum accepted values for these errors.

The paper is organized as follows. In the following section the concept of the Intelligent Space (IS) will be introduced and the dynamic model used in the tracking control problem for mobile robots together with the CS structure, and a formal description of the artificial potential field method is given. Section III presents the development method for the PI-FCs as special 2-DOF controllers. Section IV provides simulation results to validate the method. Since in the case of mobile robot control there can appear often parametric variations of the controlled plant, it is necessary to perform the sensitivity analysis with this respect, and Section V offers sensitivity models. Section VI is focused on the conclusions.

II. INTELLIGENT SPACE. DYNAMIC MODEL OF MOBILE ROBOTS. TRACKING CONTROL SYSTEM STRUCTURE

The IS represents a space (room, corridor or street), which has distributed sensory intelligence (various sensors, such as cameras and microphones with intelligence, haptic devices to manipulate in the space) and it is equipped with actuators [15]. Actuators (speakers, screens, pointing devices, switches or robots and slave devices inside the space) are mainly used to provide information and physical support to the inhabitants. The various devices of sensory intelligence cooperate with each other autonomously, and the whole space has high intelligence [16], [17]. Each intelligent agent in the IS has sensory intelligence [18]. The intelligent agent has to operate even if the outside environment changes, so it needs to switch its roles autonomously. The agent knows its role and can support man. The IS recomposes the whole space from each agent sensory information, and returns intuitive and intelligible reactions to man. In this way, the IS represents the space where man and agents can act mutually. There is an IS, which can sense and track the path of moving objects (human beings) in a limited area. There are some mobile robots controlled by the intelligent space, which can guide blind persons in this limited area. The IS tries to identify the behavior of moving objects (human beings) and tries to predict their movement in the near future. Using this knowledge, the IS can help avoiding the fixed objects and moving ones (human beings) in the IS.

The proposed assistant mobile robot is presented in Fig. 1 (a), and its base is a mobile robot platform. This platform has a tricycle kinematics and drives along arcs, determined by steer angle θ and speed v of the steered front wheel as shown in Fig. 1 (b). The robot needs plenty information about its surroundings. This information mostly provided by the IS. For the maximum safety the robot has own sensors mounted on the mobile platform. When the connection is lost with the IS, or the provided information is not reliable, then the robot uses its own sensor system.

By introducing the dynamics to the kinematic model [6] and by extending the dynamic model in [19], the result can be expressed as the following dynamic model of mobile robots with two degrees of freedom:

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{v} &= a_v \\ \dot{\theta} &= \omega \\ T_{\Sigma 1}\dot{a}_v + a_v &= k_{P1}(u_1 + d_1) \\ T_{\Sigma 2}\dot{\omega} + \omega &= k_{P2}(u_2 + d_2) \end{aligned} \quad , \qquad (1)$$

where the variables represent: (x, y) – coordinates of the center of the rear axis of the mobile robot (Fig. 1 (b)); v and a_v – forward velocity and acceleration, respectively; θ – angle between the heading direction and the x-axis; ω – angular velocity; u_1 and u_2 – control signals, proportional to the torques or to the generalized force variables; d_1 and d_2 – disturbance inputs due to the contact with the IS. The parameters in (1) are: k_{P1} and k_{P2} – gains; $T_{\Sigma 1}$ and $T_{\Sigma 2}$ – small / parasitic time constants or time constants equivalent to the cumulative effects of the dynamics of actuators, measuring devices and control equipment.

The structure of the model (1) can be illustrated by means of the block diagram of the controlled plant (Fig. 2), that outlines its dynamic subsystems having the transfer functions $H_{P1}(s)$ and $H_{P2}(s)$ of the controlled plant:

$$H_{Pi}(s) = k_{Pi}/[s(1+T_{\Sigma i}s)],\ i = \overline{1,2}, \qquad (2)$$

and its kinematic subsystem (KS).

The CS structure employed in tracking control for mobile robots with two degrees of freedom is presented in Fig. 3, where: C-1 and C-2 – forward velocity controller and angle controller, respectively; v_r and θ_r – reference inputs for the two control loops; RF-1 and RF-2 – reference filters; $\tilde{v}_r$ and $\tilde{\theta}_r$ – filtered reference inputs for the two control loops; $e_1 = \tilde{v}_r - v$ and $e_2 = \tilde{\theta}_r - \theta$ – control errors; x_r and y_r – reference positions for x and y, respectively; $e_x = x_r - x$ and $e_y = y_r - y$ – tracking errors for x and y, respectively; CB-xp and CB-yp – computation blocks providing the estimates $\hat{\dot{x}}_r$ and $\hat{\dot{y}}_r$ of the derivatives $\dot{x}_r$ and $\dot{y}_r$, respectively; $\Delta x_{r,k}$ and $\Delta y_{r,k}$ – increments of the reference positions x_r and y_r, respectively; k – index of the current sampling interval.

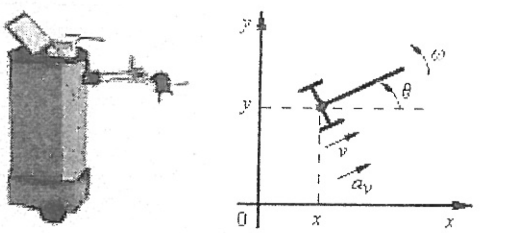

Fig. 1. Guiding and communication assistant (a); mechanical variables of mobile robots with two degrees of freedom (b)

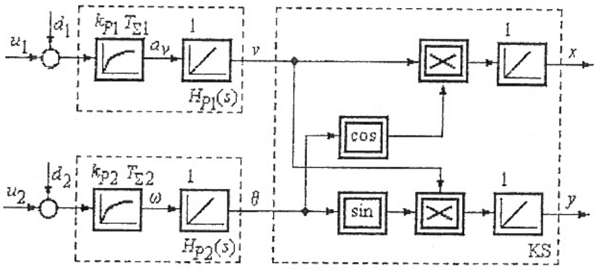

Fig. 2. Structure of simplified dynamic model as controlled plant

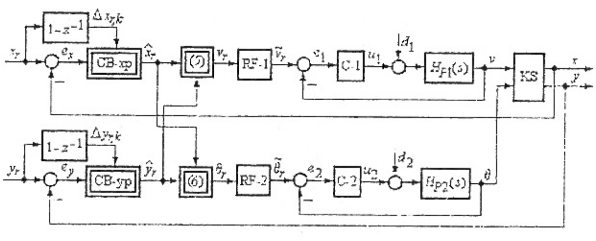

Fig. 3. Control system structure

The proposed CS structure is a cascaded one, with the two inner loops to control v and θ, and the outer loops to provide the reference inputs for the inner loops by means of the blocks CB-xp, CB-yp operating on the basis of the following algorithms:

$$\hat{\dot{x}}_{r,k} = \begin{cases} \Delta x_{r,k}/h & \text{if } |e_{x,k}| \le \varepsilon_x \\ e_{x,k}/h & \text{otherwise} \end{cases}, \qquad (3)$$

$$\hat{\dot{y}}_{r,k} = \begin{cases} \Delta y_{r,k}/h & \text{if } |e_{y,k}| \le \varepsilon_y \\ e_{y,k}/h & \text{otherwise} \end{cases}. \qquad (4)$$

where h stands for the sampling interval, and the parameters $\varepsilon_x > 0$ and $\varepsilon_y > 0$ represent the maximum accepted absolute values of the tracking errors corresponding to the positions x and y; the values of ε_x and ε_y must be specified by the CSs specialist as function of the desired CS performance.

The blocks CB-xp and CB-yp play the role of derivative estimators and, as it can be seen in (3), (4) and Fig. 3, the feedback in terms of x and y (in the outer control loops) operates only when the absolute values of the tracking errors e_x and e_y exceed the values of ε_x and ε_y, respectively.

For the computation of the reference inputs v_r and θ_r for the two inner control loops, the first two equations in (1) can be used, modified as follows:

$$v_r = \sqrt{(\hat{\dot{x}}_r)^2 + (\hat{\dot{y}}_r)^2}, \qquad (5)$$

$$\theta_r = \tan^{-1}(\hat{\dot{y}}_r / \hat{\dot{x}}_r), \qquad (6)$$

and (5), (6) characterize the nonlinear blocks in Fig. 3.

The generation of the reference trajectory (x_r, y_r) for the CS structure in Fig. 3 is performed off-line and is based on drawing a virtual potential field that ensures the obstacle avoidance. The application of the artificial potential field method [14] is based on the construction of a local harmonic potential field $\Psi(x, y)$ that satisfies the Laplace equation (7):

$$\nabla^T \cdot \nabla\Psi(x,y) = \partial^2\Psi(x,y)/\partial x^2 + \partial^2\Psi(x,y)/\partial y^2 = 0\,, \qquad (7)$$

and the solution of (7) gives the potential of a singular point of strength q at (0, 0):

$$\Psi(x,y) = -(q/2)\cdot \ln(x^2 + y^2)\,, \qquad (8)$$

and the associated gradient $\rho(x, y)$:

$$\rho(x,y) = -\mathrm{grad}\Psi(x,y) = [q/(x^2+y^2)](x,y)^T = (\rho_x,\rho_y) \in R^2\,. \qquad (9)$$

The reference trajectory (x_r, y_r) will be obtained by the integration the following equations of motion in terms of using adequate numerical techniques:

$$\begin{aligned} \dot{x}_r &= v_r[\rho_x / \sqrt{(\rho_x)^2 + (\rho_x)^2}]\cos\theta_r \\ \dot{y}_r &= v_r[\rho_y / \sqrt{(\rho_x)^2 + (\rho_x)^2}]\sin\theta_r \end{aligned}. \qquad (10)$$

The model and CS structure proposed here are quite different from the models and control structures using the reference and actual posture determined by (x, y, θ) [20] because θ is in fact not controlled directly, but it results with acceptable precision since the tracking at the level of x and y will be improved.

III. DEVELOPMENT METHOD FOR PI-FUZZY CONTROLLERS CONSIDERED AS 2-DOF CONTROLLERS

The structure of the standard version of PI-FC with integration of the control signal is presented in Fig. 4 for both PI-FCs playing the role of C-1 and C-2 in Fig. 3. This structure corresponds to a particular case of 2-DOF controllers for [21], [22], and it is based on adding the dynamics to the basic fuzzy controllers without dynamics FC_i, $i=\overline{1,2}$, by the numerical differentiation of the control error $e_{i,k}$ as the increment of control error, $\Delta e_{i,k} = e_{i,k} - e_{i,k-1}$, and the numerical integration of the increment of control signal, $\Delta u_{i,k} = u_{i,k} - u_{i,k-1}$. The other elements in Fig. 4 are: $r_i \in \{v_r, \theta_r\}$ – the reference inputs; $\tilde{r}_i \in \{\tilde{v}_r, \tilde{\theta}_r\}$ – the filtered reference inputs; $y_i \in \{v, \theta\}$ – the controlled outputs.

The development of the PI-FCs starts from the development of two linear PI controllers. In the case of plants with the transfer functions in the forms (2) the use of PI controllers having the transfer function (11):

$$H_{Ci}(s) = \frac{k_{ci}}{s}(1 + sT_{ci}),\ i = \overline{1,2}\,, \qquad (11)$$

with the gains k_{ci} and the integral time constants T_{ci}, tuned in terms of the ESO method [13], can ensure good CS performance with the controllers tuned in terms of (12):

$$k_{ci} = \frac{1}{\sqrt{\beta_i^3}\,T_{\Sigma i}^2 k_{Pi}}\,,\ T_{ci} = \beta_i T_{\Sigma i}\,,\ i = \overline{1,2}\,, \qquad (12)$$

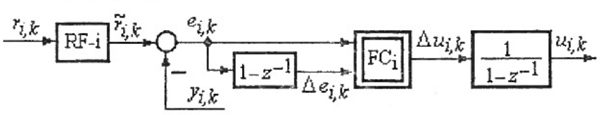

Fig. 4. Structure of standard PI-FCs as special 2-DOF controllers, $i = \overline{1,2}$

where β_i, $i=\overline{1,2}$, represent design parameters, only one for each controller.

By using (2), (11) and (12), the closed-loop transfer functions with respect to the reference inputs will obtain their optimal expressions $H_i(s)_{opt}$:

$$H_i(s)_{opt} = \frac{1 + \beta_i T_{\Sigma i} s}{\sqrt{\beta_i^3}\,T_{\Sigma}^3 s^3 + \sqrt{\beta_i^3}\,T_{\Sigma i}^2 s^2 + \beta_i T_{\Sigma i} s + 1}\,,\ i = \overline{1,2}\,, \qquad (13)$$

and the open-loop transfer functions, $H_{0i}(s)$, will obtain also the optimal expressions:

$$H_{0i}(s)_{opt} = \frac{1 + \beta_i T_{\Sigma i} s}{\sqrt{\beta_i^3}\,T_{\Sigma i}^2 s^2 (1 + T_{\Sigma i} s)}\,,\ i = \overline{1,2}\,. \qquad (14)$$

The ESO method represents a generalization of Kessler's Symmetrical Optimum method described in [23]. The ESO method performs indeed an optimization by guaranteeing the maximum value of the phase margin for constant controlled plant parameters [13]. Furthermore, the ESO method guarantees a minimum phase margin in the case of variable k_{Pi}.

By the choice of the design parameters β_i in the domain $\beta_i \in\ ]1, 20[$, the CS performance indices σ_1 – overshoot, $\hat{t}_s = t_s / T_{\Sigma i}$ – settling time, $\hat{t}_1 = t_1 / T_{\Sigma i}$ – first settling time and φ_r – phase margin can be accordingly modified and a compromise to these indices can be reached by using the diagrams in Fig. 5. These diagrams have been obtained by processing (13) and (14).

The CS performance indices can be corrected by:

- either suppressing the effect of the zero in the open-loop transfer functions, by adding the reference filters RF-1 and RF-2 having the transfer functions $H_{F1}(s)$ and $H_{F2}(s)$, respectively:

$$H_{RFi}(s) = \frac{1}{1 + \beta_i T_{\Sigma i} s}\,,\ i = \overline{1,2}\,; \qquad (15)$$

- or suppressing the effect of the zero and of the complex conjugated poles in the closed-loop transfer functions, by adding other version of reference filters:

$$H_{RFi}(s) = \frac{1 + (\beta_i - \sqrt{\beta_i})T_{\Sigma i} s + \beta_i T_{\Sigma i}^2 s^2}{(1 + \beta_i T_{\Sigma i} s)[1 + (\beta_i - \sqrt{\beta_i} - 1)T_{\Sigma i} s]}\,,\ i = \overline{1,2}\,. \qquad (16)$$

For the development of the PI-FCs the continuous PI controllers (11) are discretized resulting in the discrete-time equations of the PI quasi-continuous digital controllers expressed in their incremental versions (17):

$$\begin{aligned} \Delta u_{i,k} &= K_{Pi}\,\Delta e_{i,k} + K_{Ii}\, e_{i,k} = \\ &= K_{Pi}(\Delta e_{i,k} + \lambda_i \cdot e_{i,k}),\ i = \overline{1,2}, \end{aligned} \qquad (17)$$

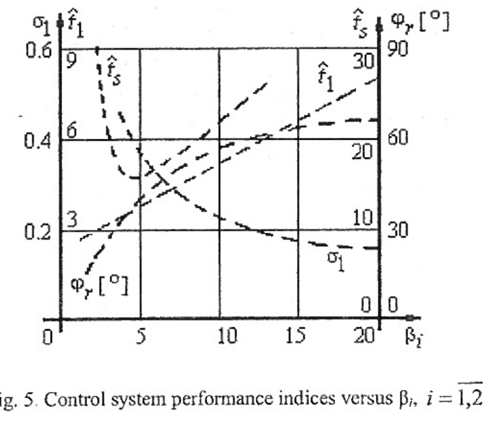

Fig. 5. Control system performance indices versus β_i, $i = \overline{1,2}$

where the parameters $\{K_{Pi}, K_{Ii}, \lambda_i\}$ depend on $\{k_{ci}, T_{ci}\}$:

$$K_{Pi}=k_{ci}T_{ci}[1-(h/2T_{ci})],\ K_{Ii}=k_{ci}h,$$
$$\lambda_i=K_{Ii}/K_{Pi}=2h/(2T_{ci}-h),\ i=\overline{1,2}. \quad (18)$$

For the basic fuzzy controllers without dynamics FC_i, $i=\overline{1,2}$ (Fig. 5), the fuzzification can be solved in the initial phase as follows (Fig. 6): for the input variables, $e_{i,k}$ and $\Delta e_{i,k}$, five linguistic terms with regularly distributed triangular type membership functions having an overlap of 1 are chosen; for the output variable $\Delta u_{i,k}$, seven linguistic terms with non-regularly distributed singleton type membership functions are chosen. Other shapes of membership functions can contribute to CS performance enhancement.

The parameters of the PI-FCs are $\{B_{ei}, B_{\Delta ei}, B_{\Delta ui}, m_i, n_i, p_i\}$. The first three parameters are in correlation with the shapes of the membership functions of the linguistic terms corresponding to the input and output linguistic variables. The parameters $m_i < n_i < p_i \in]0, 2[$ have been added to the standard version of PI-FCs for the sake of performance enhancement by alleviating the overshoot. The inference engine of the blocks FC_i employs the Mamdani's MAX-MIN compositional rule of inference assisted by complete rule bases expressed as decision tables (Table I).

For the presented version of PI-FCs the centre of gravity method is used for defuzzification in the blocks FC_i.

The development method for the two PI-FCs playing the roles of the controllers C-1 and C-2 in Fig. 3 consists of the following development steps:

- Step 1: Express the last two equations in the simplified dynamic model (1) of the mobile robot in terms of the mathematical models of the subsystems of the controlled plant with the transfer functions in (2), $H_{P1}(s)$ and $H_{P2}(s)$, and compute the parameters k_{P1}, k_{P2}, $T_{\Sigma 1}$ and $T_{\Sigma 2}$.
- Step 2: Choose the values of the design parameters β_1 and β_2 as function of the desired CS performance indices by using the diagrams in Fig. 5 and the maximum accepted absolute values ε_x and ε_y of the tracking errors corresponding to the positions x and y, respectively.
- Step 3: Tune the parameters of the continuous PI controllers $\{k_{c1}, T_{c1}\}$ (for C-1) and $\{k_{c2}, T_{c2}\}$ (for C-2) by (12).

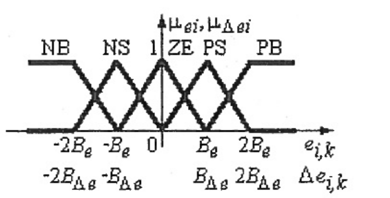

Fig. 6. Membership functions of FC_i, $i=\overline{1,2}$

TABLE I

DECISION TABLE OF STANDARD FC_i, $i=\overline{1,2}$

$\Delta e_{i,k} \backslash e_{i,k}$	NB	NS	ZE	PS	PB
PB	ZE	PS	PM	PB	PB
PS	NS	ZE	PS	PM	PB
ZE	NM	NS	ZE	PS	PM
NS	NB	NM	NS	ZE	PS
NB	NB	NB	NM	NS	ZE

- Step 4: Add to the CS structure the reference filters RF-1 and RF-2 in the versions (15) or (16).
- Step 5: Choose a sufficiently small sampling period, h, required by quasi-continuous digital control and take into account the presence of zero-order hold blocks.
- Step 6: Discretize the two continuous PI controllers and compute the parameters of the two quasi-continuous digital PI controllers, $\{K_{P1}, K_{I1}, \lambda_1\}$ and $\{K_{P2}, K_{I2}, \lambda_2\}$, by (18).
- Step 7: Set the values of the parameters m_i, n_i, p_i and B_{ei} of the PI-FCs and apply (19), resulted from the modal equivalence principle [24], to obtain the values of the parameters $B_{\Delta ei}$ and $B_{\Delta ui}$:

$$B_{\Delta ei}=\lambda_i B_{ei},\ B_{\Delta ui}=K_{Ii}B_{ei},\ i=\overline{1,2}. \quad (19)$$

The parameters of the basic linear PI controllers (11), $\{k_{c1}, T_{c1}, k_{c2}, T_{c2}\}$, are taken into consideration in the values of the parameters $B_{\Delta ei}$ and $B_{\Delta ui}$ by applying these seven development steps.

Besides setting the parameters m_i, n_i, p_i and B_{ei} in accordance with the user's experience in controlling the plant, the choice of the inference method and of the defuzzification method represents the user's option as well.

IV. CASE STUDY. DIGITAL SIMULATION RESULTS

The simulation results are obtained by accepting that the simplified dynamic model (1) characterizes the mobile robot with two degrees of freedom. The considered case study corresponds to the following values of the parameters $k_{P1}=k_{P2}=1$ and $T_{\Sigma 1}=T_{\Sigma 2}=1$ s.

There is considered an experiment with different placements of three obstacles placed in the points (7, 2), (5, 7) and (1, 2) having the potentials 0.2, 0.2 and 0.1, and the initial position of the robot is (10, 1). The goal, representing the desired final position, is placed in the point (1, 11), with the potential –1. For the considered obstacles and robot position the application of the artificial potential method leads to the vector field and to the reference trajectory (x_r, y_r) presented in Fig. 7.

The CS structure is that presented in Fig. 3. The PI-FCs playing the roles of C-1 and C-2 are developed in terms of the method presented in the previous Section, and the reference filters (15) are used. For the accepted case study, in the conditions of $\beta_1=\beta_2=4$ and $h=0.1$ s, the parameters of the PI quasi-continuous digital controllers will be $K_{P1}=K_{P2}=0.4938$, $K_{I1}=K_{I2}=0.0125$, $\lambda_1=\lambda_2=0.0253$. Setting $B_{e1}=0.3$, $B_{e2}=0.45$ and $m_1=m_2=0.8$, $n_1=n_2=1.2$, $p_1=p_2=1.4$, the values of the other parameters of the PI-FCs will be $B_{\Delta e1}=0.0076$, $B_{\Delta u1}=0.0037$, $B_{\Delta e2}=0.0114$ and $B_{\Delta u2}=0.0056$.

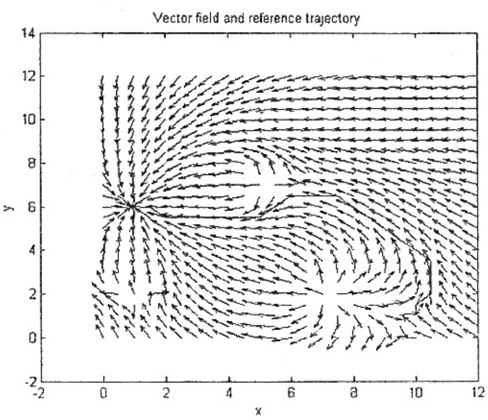

Fig. 7. Vector field and reference trajectory

For the accepted experiment different disturbance inputs d_1 and d_2 have been applied having the variations (20):

$$
\begin{aligned}
d_i = {} & \delta_i\sigma(t-110) - \delta_i\sigma(t-220) - \delta_i\sigma(t-440) + \\
& + \delta_i\sigma(t-550) + \delta_i\sigma(t-770) - \delta_i\sigma(t-880), \\
& t \in [0, 1200\,]\,\mathrm{s},\ i = \overline{1,2}\,,
\end{aligned}
\tag{20}
$$

where σ – the unit step signal, δ_1 = 0.006 and δ_2 = 0.09, and these variations are acceptable to model the contact of the robot with the IS.

In the conditions of these disturbance inputs and of the reference inputs computed off-line by using (3) … (6), Fig. 7, the synthesis of the digital simulation results for the CS with PI-FCs is presented as follows: the reference trajectory (dotted line) and the actual trajectory (continuous line) in Fig. 8, the variations of the control signal u_1 in Fig. 9, and the variations of the control signal u_2 in Fig. 10.

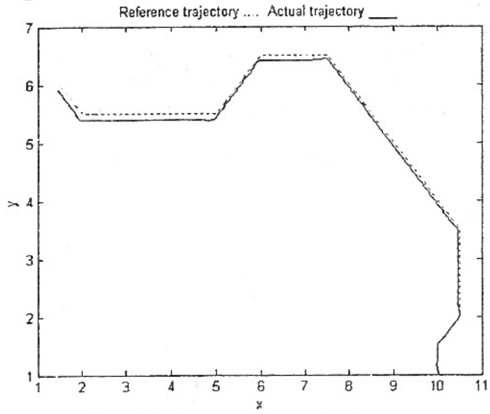

Fig. 8. Reference and actual trajectory

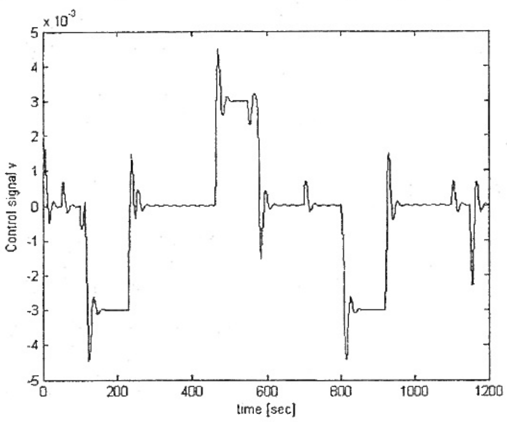

Fig. 9. Control signal u_1 versus time

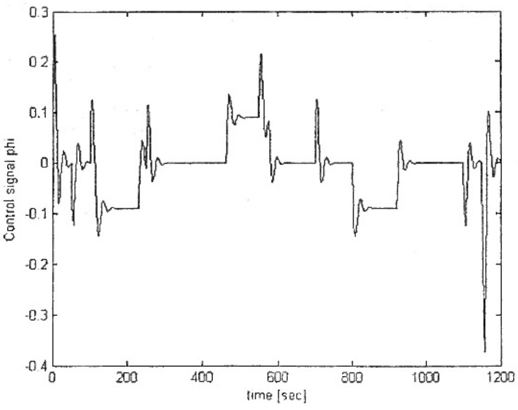

Fig. 10. Control signal u_2 versus time

V. SENSITIVITY MODELS

The variation of controlled plant parameters (k_{Pi} and $T_{\Sigma i}$, $i=\overline{1,2}$) due to the change of the steady-state operating points or to other conditions leads to additional motion of the CSs. This motion is usually undesirable under uncontrollable parametric variations. Therefore, for the development of the fuzzy controllers to alleviate the effects of parametric disturbances it is necessary to perform the sensitivity analysis with respect to the parametric variations of the controlled plant. The sensitivity models (SMs) enable the sensitivity study of the fuzzy control systems accepted in certain conditions (for example, [24]) to be approximately equivalent with the linear CSs (with the continuous-time PI controllers). In this context, it is necessary to obtain the SMs of the two linear CSs, the inner control loops in Fig. 3 with the PI controllers (11) playing the roles of C1 and C2. The SMs of the fuzzy control systems will be identical to the SMs of the linear CSs. The only difference between these SMs is in the generation of the nominal trajectory of the CSs involved, by using either the PI-FCs or the linear PI controllers.

For deriving the SMs it is necessary to obtain firstly the state mathematical models (MMs) of the CP. By considering the state variables x_{1i} (the outputs of the first-order elements) and x_{2i} (the outputs of the integral elements) corresponding to $H_{Pi}(s)$ as in Fig. 2, the state MMs of the controlled plant will be (21):

$$
\begin{aligned}
& \dot{x}_{1i}(t) = x_{2i}(t), \\
& \dot{x}_{2i}(t) = -(1/T_{\Sigma i})x_{2i}(t) + (k_{Pi}/T_{\Sigma i})u_i(t) + \\
& \qquad + (k_{Pi}/T_{\Sigma i})d_i(t), \\
& y_i(t) = x_{1i}(t),\ i=\overline{1,2}\,.
\end{aligned}
\tag{21}
$$

Then, the state MMs of the linear PI controllers can be expressed in their parallel forms (22):

$$
\begin{aligned}
& \dot{x}_{3i} = (1/T_{ci})e_i, \\
& u_i = k_{Ci}(x_{3i} + e_i),\ i=\overline{1,2}\,,
\end{aligned}
\tag{22}
$$

where x_{3i} are the outputs of the integral components of the PI controllers (in parallel form).

The original linear PI controllers are tuned in terms of (12) by considering the nominal values of controlled plant parameters, k_{Pi0} and $T_{\Sigma i0}$, $i=\overline{1,2}$. So, the state MMs of the PI controllers (11) will be transformed into:

$$
\begin{aligned}
& \dot{x}_{3i} = [(1/\beta_i T_{\Sigma i0})]e_i, \\
& u_i = [1/(\sqrt{\beta_i}\,k_{Pi0}T_{\Sigma i0})](x_{3i} + e_i),\ i=\overline{1,2}\,.
\end{aligned}
\tag{23}
$$

The state MMs of the closed-loop systems can be obtained by connecting the MMs (21) and (23):

$$
\begin{aligned}
& \dot{x}_{1i}(t) = x_{2i}(t), \\
& \dot{x}_{2i}(t) = -[k_{Pi}/(\sqrt{\beta_i}\,k_{Pi0}T_{\Sigma i0}T_{\Sigma i})]x_{1i}(t) - (1/T_{\Sigma i})x_{2i}(t) + \\
& \qquad + [k_{Pi}/(\sqrt{\beta_i}\,k_{Pi0}T_{\Sigma i0}T_{\Sigma i})]x_{3i}(t) + \\
& \qquad + [k_{Pi}/(\sqrt{\beta_i}\,k_{Pi0}T_{\Sigma i0}T_{\Sigma i})]r_i(t) + (k_{Pi}/T_{\Sigma i})d_i(t), \\
& \dot{x}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]x_{1i}(t) + [1/(\beta_i T_{\Sigma i0})]r_i(t), \\
& y_i(t) = x_{1i}(t),\ i=\overline{1,2}\,.
\end{aligned}
\tag{24}
$$

For (24) there can be derived the sensitivity functions $\{\lambda_{1i}, \lambda_{2i}, \lambda_{3i}\}$ and the output sensitivity functions, σ_i:

$$
\begin{aligned}
& \lambda_{ji}(t) = [\partial x_{ji}(t)/\partial\alpha]_{\alpha 0}, \\
& \sigma_i(t) = [\partial y_i(t)/\partial\alpha]_{\alpha 0},\ j=\overline{1,3}\,,\ i=\overline{1,2}\,,
\end{aligned}
\tag{25}
$$

where "0" stands for the nominal values of the controlled plant parameters, and $\alpha \in \{k_{Pi}, T_{\Sigma i}\}$.

Accepting the dynamic regimes characterized by the step modifications of the reference inputs r_i for $d_i(t)=0$, or the step modifications of the disturbance inputs d_i for $r_i(t)=0$, there can be derived four sets of SMs by computing the partial derivatives with respect to k_{Pi} and $T_{\Sigma i}$ in (24). The two SMs with respect to the variations of k_{Pi}, the step modifications of r_i, and $d_i(t)=0$, are expressed in (26):

$$\begin{aligned}
&\dot{\lambda}_{1i}(t) = \lambda_{2i}(t), \\
&\dot{\lambda}_{2i}(t) = -[1/(\sqrt{\beta_i}T_{\Sigma i0}^2)]\lambda_{1i}(t) - (1/T_{\Sigma i0})\lambda_{2i}(t) + \\
&\quad + [1/(\sqrt{\beta_i}T_{\Sigma i0}^2)]\lambda_{3i}(t) - [1/(\sqrt{\beta_i}k_{Pi0}T_{\Sigma i0}^2)]x_{1i0}(t) + \\
&\quad + [1/(\sqrt{\beta_i}k_{Pi0}T_{\Sigma i0}^2)]x_{3i0}(t) + [1/(\sqrt{\beta_i}k_{Pi0}T_{\Sigma i0}^2)]r_{i0}(t), \\
&\dot{\lambda}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]\lambda_{1i}(t), \\
&\sigma_i(t) = \lambda_{1i}(t), \; i = \overline{1,2}.
\end{aligned} \tag{26}$$

VI. CONCLUSIONS

The paper proposes a new development method for PI-FCs considered as 2-DOF controllers, to solve the tracking problem for wheeled mobile robots with two degrees of freedom operating in the Intelligent Space.

The controllers can be implemented as tracking controllers in a cascaded CS structure using the off-line generation of the reference inputs by means of the artificial potential field method for obstacle avoidance. In addition, the paper proposes a simplified dynamic model of the mobile robots with two degrees of freedom.

The development method for the tracking controllers has been validated with good results on the simplified dynamic model by performing a digitally simulated experiment.

The sensitivity models derived in the paper (only a part of them being presented here) enable the sensitivity analysis with respect to the parametric variations of the controlled plant.

The paper proves the ability of the presented tracking controllers in coping with the situations of local tracking control involving relatively small initial tracking errors, not arbitrary ones as it is the situation of global tracking problem. The results presented in the paper show that the proposed controllers can be applied in simpler or more complex control problems such as path following and point stabilization.

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