Artificial Potential Method for Control of a Tentacle Manipulator

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***Abstract* — The control problem of the spatial tentacle manipulator is presented. In order to avoid the difficulties generated by the complexity of the nonlinear integral - differential model, the control problem is based on the artificial potential method. It is shown that the control of a tentacle robot to a desired position is possible if the artificial potential is a potential functional whose point of minimum is attractor of this dissipative controlled system. Then, the method is used for constrained motion in an environment with obstacles.**

Numerical simulations for spatial and planar tentacle models are presented in order to illustrate the efficiency of the method.

Keywords* — *potential, control, hyper-redundant system.

I. INTRODUCTION

A tentacle manipulator is a hyper-redundant manipulator or hyper-degree-of-freedom manipulator and, over the past several years, there has been a rapidly expanding interest in the study and construction of them.

The control of these systems is very complicated and a great number of researchers tried to offer solutions for this difficult problem. In [1] it was analysed the control by cables or tendons meant to transmit forces to the elements of the arm in order to closely approximate the arm as a truly continuous backbone. Also, Mochiyama has investigated the problem of controlling the shape of an HDOF rigid-link robot with two-degree-of-freedom joints using spatial curves [7], [8]. Important results were obtained by Chirikjian and Burdick [3]-[6] that laid the foundations for the kinematic theory of hyper-redundant robots. Their results are based on a "backbone curve" that captures the robot's macroscopic geometric features. The inverse kinematic problem is reduced to determining the time varying backbone curve behaviour. New methods for determining "optimal" hyper-redundant manipulator configurations based on a continuous formulation of kinematics are developed. In [2], Gravagne analysed the kinematic model of "hyper-redundant" robots, known as "continuum" robots. Robinson and Davies [9] present the "state of art" of continuum robots, outline their areas of application and introduce some control issues.

In other papers [10, 11, 12] several technological solutions for actuators used in hyper-redundant structures are presented and conventional control systems are introduced.

All these papers treat the control problem from the kinematic point of view and few researchers focus their efforts on the dynamic problem of these systems. The dynamic models of these manipulators are very complicated. In [13] is proposed a dynamic model for hyper-redundant structures as an infinite degree-of-freedom continuum model and some computed torque control systems are introduced. In [14] a dynamic model for an ideal planar tentacle system is presented and optimal control solutions are discussed. In [15] a sequential distributed control is proposed for a tentacle manipulator actuated by electrorheological fluids.

The difficulty of the dynamic control is determined by integral-partial-differential models with high nonliniarities that characterise the dynamic of these systems.

In [21] the dynamic model for 3D space is inferred and a control law based the energy of the system is analysed.

In this paper the method of artificial potential is developed for these infinite dimensional systems. In order to avoid the difficulties associated with the dynamical model, the control law is based only on the gravitational potential and a new artificial potential. It is shown that to drive the tentacle robot to a desired position it is possible if the artificial potential is a potential functional whose point of minimum is attractor for this dissipative controlled system. Also, this method is used for constrained motion in the environment with obstacles.

The paper is organised as follows: section 2 reviews the basic principles of a tentacle manipulator; section 3 presents the general model of this system; section 4 introduces the unconstrained control problem; section 5 discusses the constrained control problem; section 6 verifies by computer simulations the control laws for a 2D and 3D tentacle manipulator.

II. BACKGROUND

We will consider an ideal tentacle arm, with a uniformly distributed mass and torque, with ideal flexibility that can take any arbitrary shape (Fig. 1). Technologically, we will analyse a backbone structure with peripheral cells that can determine the shape of the arm by an appropriate control. We will neglect friction and structural damping.

The essence of the tentacle model is a *3*-dimensional backbone curve *C* that is parametrically described by a vector $r(s) \in \mathbf{R}^3$ and an associated frame $\Phi(s) \in \mathbf{R}^{3\times 3}$ whose columns create the frame bases (Fig. 2a). The independent parameter s is related to the arc-length from the origin of the curve. We denote by *l* the total length of the arm on curve *C*.

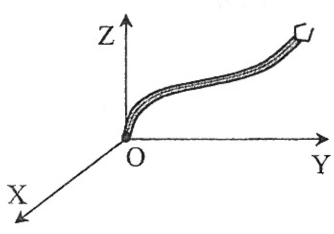

Figure 1. Tentacle model.

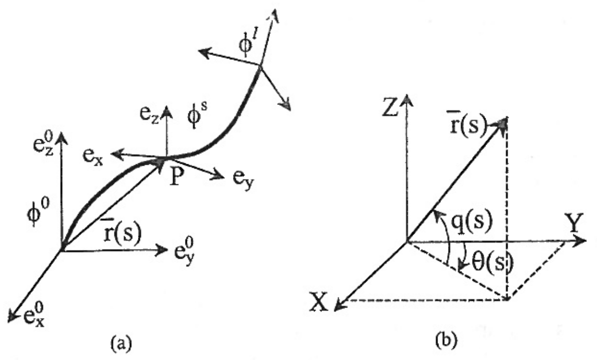

Figure 2. Tentacle system parameters.

The position of a point s on curve C is defined by the position vector,

$$\bar{r} = \bar{r}(s) \tag{1}$$

where $s \in [0, l]$. For a dynamic motion, the time variable will be introduced, $\bar{r} = \bar{r}(s,t)$.

We used a parameterisation of the curve C based upon two "continuous angles" $\theta(s)$ and $q(s)$ [3]-[6], (Fig. 2b). At each point $\bar{r}(s,t)$, the robot's orientation is given by a right-handed orthonormal basis vector $\{\bar{e}_x, \bar{e}_y, \bar{e}_z\}$ and its origin coincides with point $\bar{r}(s,t)$, where the vector e_x is tangent and e_z is orthogonal to the curve C.

The position vector on curve C is given by

$$\bar{r}(s,t) = [x(s,t) \;\; y(s,t) \;\; z(s,t)]^T \tag{2}$$

where

$$x(s,t) = \int_0^s \sin\theta(s',t)\cos q(s',t)ds' \tag{3}$$

$$y(s,t) = \int_0^s \cos\theta(s',t)\cos q(s',t)ds' \tag{4}$$

$$z(s,t) = \int_0^s \sin q(s',t)ds' \tag{5}$$

with $s' \in [0,s]$. We can adopt the following interpretation [2, 6]: at any point s the relations (2)-(5) determine the current position and the matrix Φ^s contains the robot's orientation, and the robot's shape is defined by the behaviour of functions $\theta(s)$ and $q(s)$. The robot "grows" from the origin by integrating to get $\bar{r}(s,t)$.

The velocity components are

$$v_x = \int_0^s \left(-\dot{q}'\sin q'\sin\theta' + \dot{\theta}'\cos q'\cos\theta'\right)ds' \tag{6}$$

$$v_y = \int_0^s \left(-\dot{q}'\sin q'\cos\theta' - \dot{\theta}'\cos q'\cos\theta'\right)ds' \tag{7}$$

$$v_z = \int_0^s \dot{q}\cos q' ds' \tag{8}$$

For an element dm, kinetic and potential energy will be

$$dT = \frac{1}{2}dm \cdot v^2 \tag{9}$$

$$dV = dm \cdot g \cdot z \tag{10}$$

where

$$dm = \rho ds \tag{11}$$

From (8)-(11) we obtain,

$$T = \frac{1}{2}\rho\int_0^l\left(\left(\int_0^s\left(-\dot{q}\sin q'\sin\theta' + \dot{\theta}'\cos q'\cos\theta'\right)ds'\right)^2 + \right.$$

$$+\left(\int_0^s\left(-\dot{q}'\sin q'\cos\theta' - \dot{\theta}'\cos q'\sin\theta'\right)ds'\right)^2 +$$

$$\left. + \left(\int_0^s \dot{q}'\cos q' ds'\right)^2\right)ds \tag{12}$$

$$V = \rho g\int_0^l\int_0^s \sin q' ds' ds \tag{13}$$

We will consider $F_\theta(s,t), F_q(s,t)$ the distributed forces on the length of the arm that determine motion and orientation in the θ - plane, q - plane. From [21] we obtain the mechanical work,

$$L = \int_0^l\int_0^t\left(F_\theta(s,\tau)\dot{\theta}(s,\tau) + F_q(s,\tau)\dot{q}(s,\tau)\right)d\tau ds \tag{14}$$

where $\dot{\theta}$, $\dot{q}$ denote

$$\dot{\theta}(s,t) = \frac{\partial\theta}{\partial t}(s,t) \tag{15}$$

$$\dot{q}(s,t) = \frac{\partial q}{\partial t}(s,t) \tag{16}$$

The energy-work relationship will be

$$[T(t) + V(t)] - [T(0) + V(0)] =$$

$$= \int_0^l\int_0^t\left(F_\theta(s,\tau)\dot{\theta}(s,\tau) + F_q(s,\tau)\dot{q}(s,\tau)\right)d\tau ds \tag{17}$$

where $T(t)$ and $T(0)$, $V(t)$ and $V(0)$ are the total kinetic energy and total potential energy of the system at time t and 0, respectively.

III. Dynamic System

In this paper, the manipulator model is considered as a distributed parameter system defined on a fixed spatial domain $\Omega = [0,l]$ and the spatial coordinate is denoted by s.

The dynamic model of this manipulator with hyperredundant configurations can be obtained, in general form, from Hamilton partial differential equations [13, 18] of the distributed parameter model,

$$\frac{\partial\omega(t,s)}{\partial t} = \frac{\delta H}{\delta v(t,s)} \tag{18}$$

$$\frac{\partial v(t,s)}{\partial t} = -\frac{\delta H}{\delta\omega(t,s)} + F(t,s) \tag{19}$$

where ω and ν are the generalized coordinates and momentum densities, respectively, and $\delta(\cdot)/\delta(\cdot)$ denotes a functional partial derivative.

The state of this system at any fixed time t is specified by the set $(\omega(t,s),\nu(t,s))$, where $\omega=[\theta \quad q]^T$. The set of all functions of $s\in\Omega$ that ω, ν can take on at any time is state function space $\Gamma(\Omega)$. We will consider that $\Gamma(\Omega)\subset L_2(\Omega)$. The control force is the distributed vector force along the arm

$$F=\begin{bmatrix}F_\theta & F_q\end{bmatrix}^T \tag{20}$$

A practically form of dynamical model expressed only as a function of generalised coordinates is derived by using Lagrange equations developed for infinite dimensional systems,

$$\frac{\partial}{\partial t}\left(\frac{\delta T}{\delta\dot{\theta}(t,s)}\right)-\frac{\delta T}{\delta\theta(t,s)}+\frac{\delta V}{\delta\theta(t,s)}=F_\theta \tag{21}$$

$$\frac{\partial}{\partial t}\left(\frac{\delta T}{\delta\dot{q}(t,s)}\right)-\frac{\delta T}{\delta q(t,s)}+\frac{\delta V}{\delta q(t,s)}=F_q \tag{22}$$

In [14, 21] this model has been studied and the difficulties of this complex mathematical descriptions were presented.

The great number of parameters, theoretically an infinite number of parameters, the complexity of the dynamical model make the application of the classical algorithms to obtain the control law very difficult. In much of the literature concerned with the control of these systems, the complexity of the problem is emphasized and various methods that compensate all nonlinear terms in dynamics in real time are developed in order to reduce the complexity of control systems [4-6]. Also, simplified procedures are introduced or the difficult components are neglected in order to generate a particular law for position or motion control. In all these cases, these methods require a large amount of complicated calculation so that it is difficult to implement these methods with usual level controllers. In addition, the reliability of these methods may be lost when a small error in computation or a small change in system's parameters occurs [13, 18].

In the next section, the artificial potential method is extended to the control problem of this manipulator.

IV. Unconstrained Control Problem

The artificial potential is a potential function whose points of minimum are attractors for a dissipative controlled system. It was shown [13, 16, 17, 19] that the control of robot motion to a desired point is possible if the function has a minimum in the desired point. In this section we will extend this result for the infinite dimensional model of the tentacle manipulator.

We consider that the initial state of the system is given by

$$\omega_0=\omega(0,s)=[\theta_0,\ q_0]^T \tag{23}$$

$$\nu_0=\nu(0,s)=[0,\ 0]^T \tag{24}$$

where

$$\theta_0=\theta(0,s),\ q_0=q(0,s),\ s\in[0,l] \tag{25}$$

corresponding to the initial position of the manipulator defined by the curve C_0

$$C_0:(\theta_0(s),\ q_0(s)),\ s\in[0,l] \tag{26}$$

The desired point in $\Gamma(\Omega)$ is represented by a desired position of the arm, the curve C_d,

$$\omega_d=[\theta_d,q_d]^T,\quad \nu_d=[0,\ 0]^T$$

$$C_d:(\theta_d(s),\ q_d(s)),\ s\in[0,l] \tag{27}$$

The system motion (21), (22) corresponding to a given initial state $(\omega_0,\ \nu_0)$ defines a trajectory in the state function space $\Gamma(\Omega)$.

The control problem of the manipulator means the motion control by the forces $F_\theta,\ F_q$ from the initial position C_0 to the desired position C_d.

From the viewpoint of mechanics, desired position $(\omega_d,\ \nu_d)$ is asymptotically stable if the potential function of the system has a minimum at

$$(\omega,\ \nu)(s)=(\omega_d,\ \nu_d)(s),\quad s\in[0,l] \tag{28}$$

and the system is completely damped [13, 16].

As a control problem, we will extend the result of [13] and we will change the dynamic and static mechanical properties by modifying the potential function.

We will consider the control forces,

$$F_\theta(t,s)=\frac{\delta V}{\delta\theta(t,s)}-F_{\theta d}-\frac{\delta V^*}{\delta\theta(t,s)} \tag{29}$$

$$F_q(t,s)=\frac{\delta V}{\delta\theta(t,s)}-F_{qd}-\frac{\delta V^*}{\delta q(t,s)},\ s\in[0,l] \tag{30}$$

where first terms compensate the gravitational potential, second terms assure the damping control and the third components define the new artificial potential introduced in order to assure the motion to the desired position. The minimum points of this potential must be identical with desired positions of the manipulator, as attractors of its motion. For example, the potential V^* can be selected as a functional of generalised coordinates,

$$V^*(\theta,q)=\int_\theta^l\left((\theta-\theta_d(s))^2+(q-q_d(s))^2\right)ds. \tag{31}$$

Of course, it is clear that this functional has a single minimum for

$$\theta(t,s)=\theta_d(s),\quad q(t,s)=q_d(s),\ s\in[0,l] \tag{32}$$

The potential V^* can be defined, also, as a functional of position coordinates (x,y,z),

$$V^*(x,y,z)=\int_0^l\left((x-x_d(s))^2+(y-y_d(s))^2+(z-z_d(s))^2\right)ds \tag{33}$$

where $x_d,\ y_d,\ z_d$ are obtained from (3)-(5) for $\theta(s,t)=\theta_d(s),\ q(s,t)=q_d(s),\ s\in[0,l]$.

The control law (29), (30) modifies the system potential and the Lagrange equations (21), (22) become

$$\frac{\partial}{\partial t}\left(\frac{\delta T}{\delta\dot{\theta}(t,s)}\right)-\frac{\delta T}{\delta\theta(t,s)}+\frac{\delta V^*}{\delta\theta(t,s)}=F_{\theta_d} \tag{34}$$

$$\frac{\partial}{\partial t}\left(\frac{\delta T}{\delta\dot{q}(t,s)}\right)-\frac{\delta T}{\delta q(t,s)}+\frac{\delta V^*}{\delta q(t,s)}=F_{q_d} \tag{35}$$

The force components F_{θ_d}, F_{q_d} represent the damping components of the control [12, 13, 18], and have the form

$$F_{\theta_d}(s,t)=-\int_{0}^{l}K_\theta(s,s')\dot{\theta}(s',t)ds' \tag{36}$$

$$F_{q_d}(s,t)=-\int_{0}^{l}K_q(s,s')\dot{q}(s',t)ds' \tag{37}$$

where $K_\theta(s,s')$, $K_q(s,s')$ are positive definite specified spatial weighting functions on $(\Omega\times\Omega)$. For practical reasons, the derivative components of the control have the form

$$K_\theta(s,s')=\delta(s-s')\cdot k_\theta(s) \tag{38}$$

$$K_q(s,s')=\delta(s-s')\cdot k_q(s) \tag{39}$$

and (34), (35) become

$$F_{\theta_d}(s,t)=-k_\theta(s)\cdot\dot{\theta}(s,t) \tag{40}$$

$$F_{q_d}(s,t)=-k_q(s)\cdot\dot{q}(s,t) \tag{41}$$

Theorem 1. If the potential function V^* has a single minimum at (ω_d, v_d), the motion of the system (34), (37) converges to the desired position and the desired state (ω_d, v_d) is asymptotically stable.

Proof. See Appendix 1.

The force control (36), (37) represents a very important component of the control law that ensures the damping control of the system. For the following, we will consider the particular form (40), (41) that represents the derivative components of the conventional controllers. The selection of the parameters $k_\theta(s)$, $k_q(s)$ can be obtained from the classical methods of the PD controllers. Also, in [13] it is proved that this derivative control is optimal with respect to an optimal index. We will extend this result for the distributed parameter model of the tentacle manipulator.

Consider the performance index associated to minimum energy,

$$I=\frac{1}{2}\int_{0}^{t}\int_{0}^{l}F^T(s,t)K^{-1}(s)F(s,t)dsdt \tag{42}$$

where

$$K^{-1}(s)=diag\left(k_\theta^{-1}(s),\ k_q^{-1}(s)\right) \tag{43}$$

Theorem 2. For any potential function $V^*(\omega, v)$, the control law (40), (41) is optimal in the sense that the performance index (42) is minimized.

Proof. See Appendix 2.

V. Constrained Control Problem

Let B be the region of the state space where the mechanical system motion is not admissible, its complement $\overline{B}$ is the region of admissible movements and ∂B is the boundary of B. The control problem is to determine the potential function $V^*(\theta,q)$ which would determine the motion to the desired position $(\omega_d(s), v_d(s))$, $s\in[0,l]$ and it does not penetrate the constrained area B. In the terms of the artificial potential, this means that this functional should have a single stationary point in $\overline{B}$ and grows without limit when the system penetrates the boundary ∂B.

We will consider the following artificial potential [16],

$$V^*(\theta,q)=max\{V_1^*(\theta,q),\ V_2^*(\theta,q)\} \tag{44}$$

where $V_1^*(\theta,q)$ is the artificial potential for unconstrained problem and $V_2^*(\theta,q)$ is the potential for constrained control problem.

$V_2^*(\theta,q)$ is a nonnegative, continuous functional defined in $\overline{B}$ and

$$\lim_{d\to 0} V_2^*(\theta,q)=\infty \tag{45}$$

where d is the distance between the current state (θ,q) and the boundary ∂B.

VI. Simulations

In this section, some numerical simulations are carried out on 3D and 2D tentacle manipulators.

Example 1. We consider a spatial tentacle manipulator that operates in OXYZ space. The mechanical parameters are: linear density ρ=*2.2* kg/m and the length of the arm l = *0.3* m.

The initial position of the arm is assumed to be horizontal (0Y-axis),

$$\theta(s,0)=0;\ \ q(s,0)=0;\ \ s\in[0,\ 0.3] \tag{46}$$

and the desired position is represented by a line in OXYZ frame that is defined in terms of motion parameters as

$$\theta_d(s)=\frac{\pi}{5};\ \ q_d(s)=\frac{\pi}{5};\ \ s\in[0,\ 0.3] \tag{47}$$

The unconstrained control law is given as (29), (30) where the gravitational potential V has the form [21]

$$V=\rho g\int_{0}^{l}\int_{0}^{s}\sin q'ds'ds \tag{48}$$

where $q'=q(s')$, $s'\in[0,s]$, the artificial potential is obtained from (31), (47)

$$V^*(\theta,q)=\int_{0}^{l}\left(\left(\theta(s)-\frac{\pi}{5}\right)^2+\left(q(s)-\frac{\pi}{5}\right)^2\right)ds \tag{49}$$

and the damping control components have the form (40), (41) are selected as to minimize the performance index (42) with

$$k_\theta(s)=k_q(s)=1.015,\qquad s\in[0,l] \tag{50}$$

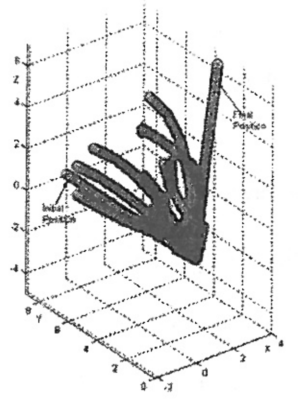

Figure 3. 3D model motion.

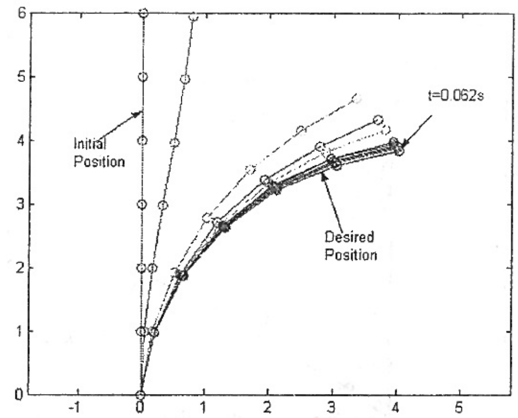

Figure 5. 2D motion for unconstrained control problem

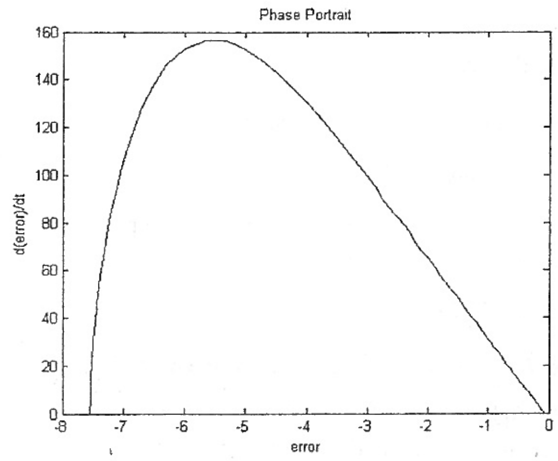

Figure 4. Phase portrait for 3D-motion.

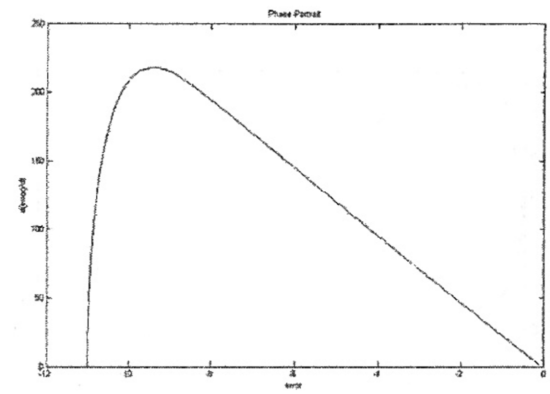

Figure 6. Phase portrait for unconstrained control problem

To simulate the dynamical model it is used the integral-differential model discussed in [21]. A discretization of the Ω – space with an increment $\Delta = 0.05$ m is used

$$s_i = i \cdot \Delta \quad i = 1, 2, \ldots, 6$$

and a MATLAB system is applied. The result is presented in Fig. 3. We see the initial position (on the OY axis), the final position and also several intermediary positions.

The phase portrait of the evolution is plotted in Fig. 4 where the error for the global system is defined as

$$e(t) = \int_0^l \left((q(s,t) - q_d(s))^2 + (\theta(s,t) - \theta_d(s))^2 \right) ds \qquad (51)$$

We see the stability of motion and error convergence to zero.

Example 2. A better understanding of the control can be obtained for 2D-tentacle arm.

We analyse now the case of a planar tentacle model in OXZ plane. The dynamic model is obtained for the case $\theta = 0$. The initial position is determined by the vertical line (OZ-axis),

$$q(s,0) = \frac{\pi}{2}, \quad s \in [0, 0.3] \qquad (52)$$

and the desired position is given by

$$q_d(s_i) = \frac{\pi}{3} - \sin\frac{\pi}{6}, \quad i = 1, 2, \ldots, 6 \qquad (53)$$

The artificial potential has the form (49) and the unconstrained law has the damping component for

$$k_q = 1.015 .$$

The results of the simulation are presented in Fig. 5 and the phase portrait is plotted in Fig. 6.

Example 3. We will consider the constrained control problem for the planar model. The initial position is the vertical position (52) and the desired position is given by

$$q_d(s_i) = \frac{\pi}{6}, \quad s \in [0, 0.3] \qquad (54)$$

A constrained area B defined in XOZ plan by the circle,

$$\partial B : (x-3)^2 + (z-6)^2 = 2.25 \qquad (55)$$

is imposed.

The artificial potential $V^*(q)$ is obtained from (44), (45) with

$$V_1^*(q) = \int_0^l \left(q(s) - \frac{\pi}{6} \right)^2 ds \qquad (56)$$

$$V_2^*(q) = \left\{ max \frac{\lambda}{d(r, \partial B)} - c, 0 \right\} \qquad (57)$$

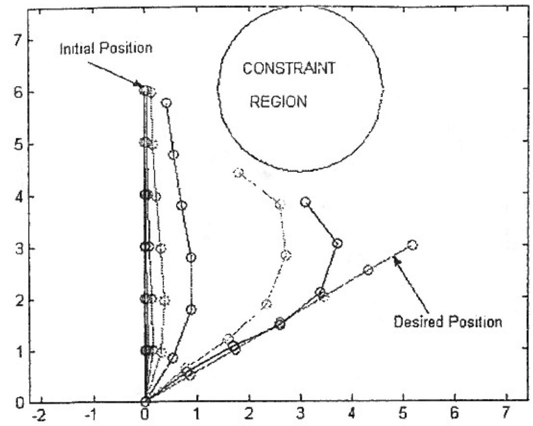

Figure 7. 2D motion for constrained control problem

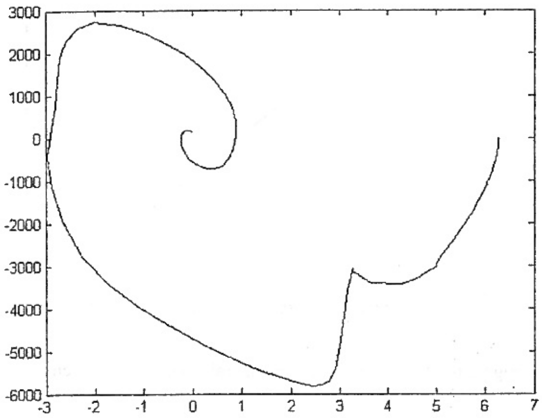

Figure 8. Phase portrait for constrained control problem

where r represents the terminal point vector of the arm and λ and c are selected as [16],

$\lambda = 1,\ c = 0.01.$

The result of the simulation is presented in Fig. 7 and the phase portrait is plotted in Fig. 8.

VII. CONCLUSIONS

The paper treats the control problem of a tentacle manipulator. In order to avoid the difficulties generated by the complexity of the nonlinear integral-differential equations that define the dynamic model of this system, the control problem is based on the artificial potential method. This method is developed for these infinite dimensional systems with unknown dynamical model except for the gravitational component.

It is shown that control of a tentacle robot to a desired position is possible if the artificial potential is a potential functional whose point of minimum is attractor of this dissipative controlled system. Also, this method is used for constrained motion in the environment with obstacles.

These results are illustrated by simulation of several tentacle models in 2D and 3D space.

APPENDIX 1

The proof is immediately [13, 16, 18, 21].

Consider a Liapunov functional based on total energy the system

$$W(t) = T(t) + V(t) \tag{58}$$

and, from (17), the derivative will be

$$\dot{W}(t) = \int_0^l \left(F_{\theta_d}(s,t)\dot{\theta}(s,t) + F_{q_d}(s,t)\dot{q}(s,t) \right) ds \tag{59}$$

By using the control (34), (35), we obtain

$$\dot{W}(t) = -\int_0^l\int_0^l \left(\dot{\theta}(s,t)K_\theta(s,s')\dot{\theta}(s',t) + \dot{q}(s,t)K_q(s,s')\dot{q}(s',t) \right) ds\, ds' \tag{60}$$

$\dot{W}(t) < 0$ (Q.E.D)

APPENDIX 2

Define the new Hamilton functional

$$H^*(\omega, \nu) = T(\omega, \nu) + V^*(\omega, \nu) \tag{61}$$

Let $\Pi(\omega, \nu, t)$ be the minimum value of the performance index (42),

$$\Pi(\omega, \nu, t) = \min_{F \in \tilde{F}} I \tag{62}$$

where $\tilde{F}$ is the domain of any admissible control functions $F = [F_\theta \ F_q]^T$. For the optimal control problem defined by the system (18), (19), (61) with the performance index (42), the dynamic programming equation will be [18].

$$\frac{\partial \Pi(\omega,\nu,t)}{\partial t} = -\min_{F \in \tilde{F}} \left\{ \int_0^l \left(\frac{\delta \Pi^T}{\delta \omega} \cdot \frac{\delta H^*}{\delta \nu(t,s)} + \frac{\delta \Pi^T}{\delta \nu} \cdot \left(-\frac{\delta H^*}{\delta \omega(t,s)} + F \right) + \frac{1}{2} F^T K^{-1} F \right) ds \right\} \tag{63}$$

The optimal control F^* that satisfies the right-hand side of this equation is

$$F^* = -K^{-1}\frac{\partial \Pi}{\partial \upsilon} \tag{64}$$

but, from [18], the optimal solution of Π is given by

$$\Pi(\omega, \nu, t) = H^*(\omega, \nu, t) \tag{65}$$

Substituting (62) in (61) and using the equation (18), it can be readily deduced that

$$F^* = -K^{-1}\frac{\delta \omega}{\delta t} \tag{66}$$

or, for the components

$$F_\theta^*(s,t) = -k_\theta^{-1}(s) \cdot \dot{\theta}(s) \tag{67}$$

$$F_q^*(s,t) = -k_q^{-1}(s) \cdot \dot{q}(s) \quad \text{Q.E.D.} \tag{68}$$

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