

Adaptive tracking control algorithm for a robot manipulators system

Mirela TRUSCA

Automation Department

Technical University of Cluj-Napoca,
St. Daicoviciu, 15, Cluj-Napoca,
ROMANIA

Mirela.Trusca@aut.utcluj.ro

Gheorghe LAZEA

Automation Department

Technical University of Cluj-Napoca,
St. Daicoviciu, 15, Cluj-Napoca,
ROMANIA

Gheorghe.Lazea@aut.utcluj.ro

Petru DOBRA

Automation Department

Technical University of Cluj-Napoca,
St. Daicoviciu, 15, Cluj-Napoca,
ROMANIA

Petru.Dobra@aut.utcluj.ro

Abstract – The case of an adaptive controllers is developed for a robot system actuated by brushed direct current motors in the presence of external disturbances and parametric uncertainties. The control scheme requires the measurements of link position and armature current for feedback. The elaborated adaptive controllers results in a closed-loop system locally stable while the all states and signals are bounded and the tracking error can be obtained as small as possible. The advantage of the presented algorithm consists in the number of parameter estimates equal to the number of unknown parameters throughout the entire mechanical system. In consequence, it is eliminated the overparametrization induced by employing the integrator backstepping technique in control of electrically driven robots. Finally, the performance of the proposed approach is illustrated in simulation examples.

I. INTRODUCTION

In the past few decades the motion control of industrial manipulators is a central issue in the robot research area. Many approaches without including the actuators dynamics have been introduced to consider the motion tracking control problem and various adaptive and robust control algorithm have been created. But the main drawback of the above control schemes consists in assuming the torque applied directly to the robot link. In consequence, for getting high-performance tracking controllers, especially in the cases of high-velocity movements and high varying loads, must be taken into account the actuator dynamics that reshape significantly the model of the robot system. The inclusion of the actuator dynamics in robot model complicates considerably the equations of motion.

Several approaches have been introduced, avoiding some parts of the above limitations by feedback linearization and singular perturbation techniques for the case of robot systems actuated by brushed direct current (BDC), brushless direct current (BLDC), switched reluctance (SR) and induction (ID) motors. Other solutions such as non-linear feedback linearizing controller [Tarn *et al.* 1991] demonstrates that a controller design including the actuator dynamics performs much better than a controller designed only on manipulator dynamics.

The rigid-link electrically driven (RLED) robot was formulated as a singular perturbation system by Taylor *et al.* and by using a time scale separation, a controller was designed based on the slow reduced-order model of the original RLED system.

Considering the general framework of the integrator backstepping technique, different integrator-backstepping-based adaptive controller were designed to overcome the

motion tracking of the electromechanical system in the presence of non-negligible electrical and mechanical actuator dynamics also in the presence of parametric uncertainties and unknown bounded disturbances.

Another proposed adaptive strategy avoid singularities in the control law by using an additional projection algorithm to estimate the value of the determinant of moment inertia resulting in more unknown parameters that normally occur in electromechanical system dynamics.

In the same manner as the phenomenon of overparametrization, the dynamic order of the resulting adaptive controller [Burg *et al.* 1996] is quite high so the computation time is increased and the speed of parameter convergence may be slower.

This paper refers the motion tracking control of robot system actuated by BDC motors which involve parametric uncertainties and external disturbances. By using the integrator backstepping technique and suitably choosing the embedded current signal, an adaptive control scheme is developed, requiring only the measurement of link position and armature current for feedback. Based on Lyapunov-like analysis, the developed adaptive controllers guarantee that for any pre-assigned attraction region the closed-loop system error can be made as small as possible. Compared with the existing control schemes in References, the proposed adaptive control scheme eliminate the need of overparametrization in the sense that the number of the parameter estimates is minimal, that is, equal to the number of unknown parameters throughout the entire electromechanical system. Hence, the control scheme developed in this study is computationally simple and easily implemented owing to the avoidance of the computation of high-order parameter update laws and the computation of augmented regressor matrices. On the other hand, only a local convergence result can be guaranteed that is a disadvantage of the adaptive control design developed here. There is a design trade-off between stronger stability results with overparametrization and a local stability result without overparametrization developed in this paper. Finally, if the model knowledge of the entire electromechanical system is well known, then a simple linear time-varying observer-based compensator can also be constructed that uses only the measurements of the link position and armature current for feedback.

II. PROBLEM DESCRIPTION

The dynamic equations for a robot system actuated by brushed direct-current motors can be described as [Tarn *et al.* 1991]:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + d_1 \quad (1)$$

$$L\dot{i} + RI + K_B\dot{q} = v + d_2 \quad (2)$$

$$\tau = K_T I \quad (3)$$

Equation (1) denotes the motion dynamics of the robot system in which $q \in R^n$ is the generalized co-ordinates, $M(q) \in R^{n \times n}$ denotes the generalized moments of inertia, the term $C(q, \dot{q})\dot{q} \in R^n$ includes the centripetal forces and Coriolis forces, $G(q) \in R^n$ denotes the gravitational forces, and $\tau \in R^n$ represents the torque inputs to the robot link produced by the BDC actuators. Equation (2) characterizes the electrical model of the motor in which $R = \text{diag}\{R_i\} \in R^{n \times n}$ is the resistance of the armature circuit, $L = \text{diag}\{L_i\} \in R^{n \times n}$ is the inductance, $K_B = \text{diag}\{K_{bi}\} \in R^{n \times n}$ is the back emf constant of the motor, $I \in R^n$ is the armature current, and $v \in R^n$ is the armature input voltage. $d_1 \in R^n$ and $d_2 \in R^n$ denote external disturbances. The last equation (3) denotes the transformation relation between the applied torque and the armature current in which $K_T = \text{diag}\{K_{ti}\} \in R^{n \times n}$ denotes the motor torque constant.

Several fundamental proprieties for the structure of the mechanical subsystem can be exploited to facilitate control system design:

P1: The inertia matrix $M(q)$ is symmetric and positive definite for every $q \in R^n$.

P2: A suitable definition of $C(q, \dot{q})$ makes matrix $(\dot{M} - 2C)$ skew-symmetric.

P3: The robot dynamics can be linearly parametrized (LP), namely, $M(q)\dot{r} + C(q, \dot{q})r + G(q) = Y_M(q, \dot{q}, r, \dot{r})\Theta_M$, for some $r \in R^n$, where $Y_M \in R^{n \times n_M}$ is a regressor matrix of known functions and the parameter $\Theta_M \in R^{n \times n_M}$ for some $n_M > 0$ is an unknown constant vector.

For simplicity of notation, let $\Theta_E = [R_1 \dots R_n, K_{b1} \dots K_{bn}, L_1 \dots L_n]^T \in R^{3n}$ denote the unknown parameter vector of the electrical subsystem. The unknown parameter values Θ_M, Θ_E and K_T are assumed to include only the constant system parameters. Therefor, it is reasonable to assume that upper and lower bounds on these unknown constants can be determined. For example, $\lambda_k I \leq K_T \leq \lambda_{K_T} I$, $L \leq \lambda_L I$ and $K_B \leq \lambda_{K_B} I$ for some constants $\lambda_k, \lambda_{K_T}, \lambda_L$ and λ_{K_B} .

The desired reference trajectory $q_d(t)$ is assumed to be bounded, and its first, second and third derivatives are also

assumed to be bounded and available for feedback. The objective in this paper is to find a dynamic feedback controller of the form:

$$\dot{w} = f(t, w, z) \quad (4)$$

$$v = g(t, w, z) \quad (5)$$

where z consists of partial state variables which are available for feedback such that for all bounded initial conditions all the variables of the closed-loop electromechanical systems (1)-(5) are bounded as well as the trajectory errors $q - q_d$ and $\dot{q} - \dot{q}_d$ should be as small as possible.

More explicitly, let Θ_M denote the estimated value of unknown mechanical parameter Θ_M , $\hat{\Theta}_T = \text{diag}\{\hat{\theta}_{ti}\}, i=1, \dots, n$, denotes the estimated value of the unknown transformation parameter K_T^{-1} and $\hat{\Theta}_E$ denotes the estimated value of the unknown electrical parameter Θ_E . The dynamic compensator (4) will contain three parameter update laws to estimate the uncertain values Θ_M, Θ_T and Θ_E , respectively. That is, the number of parameter estimates in this design will be exactly equal to the number of unknown parameters throughout the entire electromechanical system.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

Introducing the state variables $x_1 = q$ and $x_2 = \dot{q}$, the motion equations (1)-(3) can be rewritten in the following state-space representation:

$$\dot{x}_1 = x_2 \quad (6)$$

$$M(x_1)\dot{x}_2 = -C(x_1, x_2)x_2 - G(x_1) + \tau + d_1 \quad (7)$$

$$L\dot{i} = -RI - K_B x_2 + v + d_2 \quad (8)$$

Let $\bar{x}_1(t) \stackrel{\Delta}{=} q(t) - q_d(t)$ denote the position tracking error and $\bar{x}_2(t) \stackrel{\Delta}{=} p(q(t) - q_d(t)) + \dot{q}(t) - \dot{q}_d(t)$ for some constant $p > 0$ denote the filtered link tracking error. Then the error dynamic equations with respect to $\bar{x}_1(t)$ and $\bar{x}_2(t)$ are obtained as:

$$\dot{\bar{x}}_1 = -p\bar{x}_1 + \bar{x}_2 \quad (9)$$

$$M(x_1)\dot{\bar{x}}_2 = -F(q_e) - C(x_1, x_2)\bar{x}_2 + \tau + d_1 \quad (10)$$

where $q_e \stackrel{\Delta}{=} [q^T, \dot{q}^T, q_d^T, \dot{q}_d^T, \ddot{q}_d^T]^T$ and $F(q_e) \stackrel{\Delta}{=} M(q)(\ddot{q}_d - p\dot{\bar{x}}_1) + C(q, \dot{q})(\dot{q}_d - p\bar{x}_1) + G(q)$

The error dynamic equations (9) and (10) have been widely employed to solve the adaptive tracking control of robot manipulators without incorporating the actuator dynamics in which the applied torques are assumed to be applied directly to the robot links. However, since the actuator dynamics is neglected, the application of the developed control schemes in the industrial field is limited.

The applied torque $\tau = K_T I$ for the mechanical subsystem in (10) is a synthesized vector and depends on the output, the current signal $I(t)$, of the electrical subsystem. From the concept of virtually applied force in the integrator backstepping technique, let I^* denote the desired value of the current signal $I(t)$ and define:

$$\bar{x}_3 = I - I^* \quad (11)$$

the derivative of \bar{x}_3 can be computed as:

$$L\dot{\bar{x}}_3 = -RI - K_B x_2 - LI^* + v + d_2 \quad (12)$$

In the following arguments, first it is pre-assigned the desired current signal I^* and furthermore designed the actual control input signal $v(t)$ in the electrical subsystem (12) such that the produced current $I(t)$ can be driven to the desired value I^* and, in turn, $q(t)$ can further converge asymptotically to the desired trajectory $q_d(t)$. According to different measurable signals which are available in the control design a controller will be developed depending only on the measurements of q and I .

A q and I are measurable for feedback

An observer-based adaptive tracking controller is developed to solve the motion tracking problem of uncertain rigid-link electrically driven robot system that use only the measurements of link position and armature current for feedback. For this purpose let define two auxiliary function $\Psi_M = 2\gamma_1 Y_d^T \bar{x}_1$ and $\Psi_T = 2\gamma_2 \hat{\Theta}_d^T Y_d^T$ where $\gamma_1, \gamma_2 > 0$ are adaptive gains. Choose:

$$I^* = (\hat{\Theta}_T - \Psi_T) Y_d \hat{\Theta}_M - k_1 \hat{x}_2, \quad k_1 > 0 \quad (13)$$

where \hat{x}_2 is the output of the high-pass filter :

$$\dot{\eta} = (pk_0 - k_0^2) \bar{x}_1 - k_0 \eta$$

$$\hat{x}_2 = \eta + k_0 \bar{x}_1$$

Hence, after some simple manipulations and introducing $\bar{e}_2(t) = \bar{x}_2(t) - \hat{x}_2(t)$ the derivative of \bar{x}_3 in (12) can be recomputed as:

$$L\dot{\bar{x}}_3 = -Y_E \Theta_E - K_B \bar{x}_2 - L(\hat{\Theta}_T - \Psi_T) \times \quad (14)$$

$$\times Y_d \hat{\Theta}_M + k_1 L \bar{e}_2 + v + d_2$$

where

$$Y_E(\cdot) \Theta_E = RI + K_B(-p\bar{x}_1 + \dot{q}_d) + L\left((\hat{\Theta}_T - \Psi_T) \dot{Y}_d \hat{\Theta}_M + (\hat{\Theta}_T - \Psi_T) Y_d \dot{\hat{\Theta}}_M\right)$$

Here, choose the Lyapunov function candidate as:

$$W(t, x_e) = \frac{1}{2} \bar{x}_1^T \bar{x}_1 + \frac{1}{2} \bar{x}_2^T M \bar{x}_2 + \frac{1}{2} \bar{e}_2^T M \bar{e}_2 + \frac{1}{2k_1^2} \bar{x}_3^T L \bar{x}_3 + \frac{1}{2\gamma_1} (\hat{\Theta}_M - \Psi_M)^T (\hat{\Theta}_M - \Psi_M) + \frac{1}{2\gamma_2} Tr\left((\hat{\Theta}_T - \Psi_T)^T K_T (\hat{\Theta}_T - \Psi_T)\right)$$

Consider the robot system actuated by BDC (1)-(3) with $d_1(\cdot) \in L_2[0, \infty]$, $d_2(\cdot) \in L_2[0, \infty]$, given a desired trajectory $q_d(t)$. Let an observer-based adaptive tracking controller be given by:

$$\dot{\hat{\Theta}}_M = 2\gamma_1 \dot{Y}_d^T \bar{x}_1 - 2\gamma_1 p Y_d^T \bar{x}_1 + \gamma_1 Y_d^T \hat{x}_2 \quad (15)$$

$$\begin{aligned} \dot{\hat{\Theta}}_T &= \gamma_2 (-2p\bar{x}_1 + \hat{x}_2) \hat{\Theta}_M^T Y_d^T + \\ &+ 2\gamma_1 \gamma_2 \bar{x}_1 (2\dot{Y}_d^T \bar{x}_1 - 2p Y_d^T \bar{x}_1 + Y_d^T \hat{x}_2)^T Y_d^T + \\ &+ 2\gamma_2 \bar{x}_1 \hat{\Theta}_M^T \dot{Y}_d^T \end{aligned} \quad (16)$$

$$\dot{\hat{\Theta}}_E = -\frac{\gamma_3}{k_1^2} Y_E^T \bar{x}_3 \quad (17)$$

$$\dot{\eta} = (pk_0 - k_0^2) \bar{x}_1 - k_0 \eta \quad (18)$$

$$\hat{x}_2 = \eta + k_0 \bar{x}_1$$

$$v = Y_E \hat{\Theta}_E - k_1 \bar{x}_3 \quad (19)$$

where $\bar{x}_3 = I - I^*$. There exists a choice of feedback gains k_b , k_2 and k_f such that the closed-loop system guarantees that all the variables $\hat{\Theta}_M$, $\hat{\Theta}_T$, $\hat{\Theta}_E$, q , \dot{q} , η , \bar{x}_2 , I and v are bounded [Su *et al.* 1995].

Moreover, if $d_1(\cdot) \in L_2[0, \infty] \cap L_\infty[0, \infty]$ and $d_2(\cdot) \in L_2[0, \infty] \cap L_\infty[0, \infty]$ then $\lim_{t \rightarrow \infty} (q(t) - q_d(t)) = 0$ $\lim_{t \rightarrow \infty} (\dot{q}(t) - \dot{q}_d(t)) = 0$.

Differentiating $W(t)$ along the error trajectory [Kokotovic *et al.* 1986] yields that $\dot{W}(t)$ is bounded.

A local convergence result with an arbitrarily large region of attraction is achieved and the attraction region can be also arbitrarily preassigned and explicitly constructed. For unknown but unbounded external disturbances, the position tracking error of the closed-loop dynamic system can also be shown to be uniformly ultimately bounded by employing the standard projection parameter update laws.

The observer-based adaptive control law succeeded in compensating the effects of parametric uncertainty and external disturbances.

As a side result, the observer-based adaptive tracking controller can be simplified to an observer-based tracking controller when the system parameters in the entire electromechanical system are well known.

IV. SIMULATION RESULTS

Throughout simulation examples it is verified the validity of the adaptive control algorithm on the tracking control of an electrically driven robot manipulator virtually obtained in MATLAB workspace.

The dynamic equations of mechanical and electrical subsystem dynamics for a single-link direct-driven manipulator actuated by a brushed DC motor are similar to [Su *et al.* 1995].

Let the desired reference trajectory be given as $q_d(t) = 1 + \sin(t)$. In the picture below it is compared the tracking error obtained with a passivity based observer and the one that was proposed in this paper (Fig. 1).

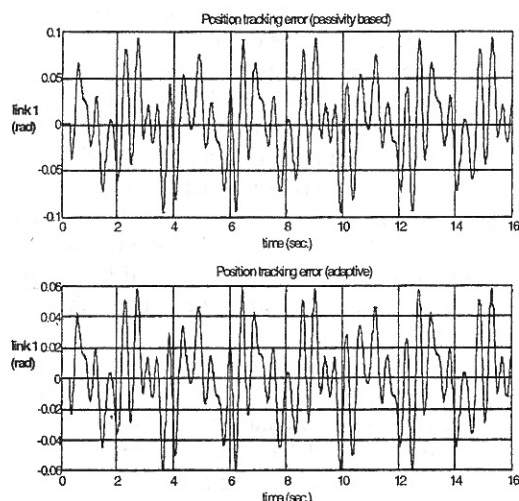


Fig. 1. Position tracking error (passivity-based first plot, adaptive second plot)

V. CONCLUSIONS

By using the integrator backstepping technique, it is developed an adaptive control scheme that requires only the link position and armature current for feedback. It is shown that the resulting closed-loop system is locally stable with an arbitrary large region of attraction and the tracking error can be made as small as possible. The attraction region can be arbitrarily preassigned and explicitly constructed. The main novelty of the developed adaptive control law is that the number of parameter estimates is exactly equal to the number of the unknown parameters throughout the entire electromechanical system.

It is worth to point out that only a local stability results without overparametrization is achieved in this study; on the contrary, global and semi-global stability results are derived in the previous control scheme with

overparametrization [Su *et al.* 1995, 1996]. Future research will focus on how to obtain a global stability result without overparametrization.

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