Multiresolution analysis in ECG signal processing

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Abstract – **The analysis of the ECG signal is extensively used as diagnostic tool to provide information on the heart function. This paper briefly introduces theory of wavelet transform and shows a few promising applications in ECG signal processing, as noise suppression , baseline wandering removal, ECG characteristic points detection. With the multiscale feature of WT's, the QRS complex can be distinguished from P or T waves, noise, baseline drift, and artifacts. Various morphologies are excited better at different scales. From these scales various segments, time widths as signal parameters can be determined more accurately.**

I. INTRODUCTION

To analyze any finite energy signal $f(t) \in \mathbf{L}_2(R)$, the continous wavelet transform (CWT) uses the dilation and translation of a single wavelet function $\psi(t)$ called mother wavelet. Specifically, if we choose the set of test functions to be

$$Q = \{\psi((t-\tau)/s), (s,\tau) \in (0,\infty) \times \mathbf{R}\} \qquad (1)$$

then we obtain the continous wavelet transform $(W_\psi f)(s,\tau)$ of the signal $f(t) \in \mathbf{L}_2(\mathbf{R})$ [1]:

$$(W_\psi f)(s,\tau) = \int_{-\infty}^{+\infty} f(t) \cdot \frac{1}{\sqrt{s}} \cdot \overline{\psi\left(\frac{t-\tau}{s}\right)} \cdot dt \qquad (2)$$

where, we have used $\overline{\psi}$ to denote the complex conjugate of ψ, and where $\psi \in \mathbf{L}_2(R)$ is an oscillatory function whose Fourier transform $\hat{\psi}(\omega)$ must satisfy:

$$C_\psi = 2\pi \int_{-\infty}^{+\infty} |\omega|^{-1} \left|\hat{\psi}(\omega)\right|^2 d\omega < \infty \qquad (3)$$

in effect if $\psi \in \mathbf{L}_1 \cap \mathbf{L}_2$, condition (3) specifies that $\psi(t)$ should have zero mean. This last condition allows for the inversion of the wavelet transform. In particular the function $f(t) \in \mathbf{L}_2(R)$ can be recovered from its transform $(W_\psi f)(s,\tau)$ by the inverse formula:

$$f(t) = C_\psi^{-1} \int_0^{+\infty} \int_R \tau^{-2} (W_\psi f)(s,\tau) \cdot \psi\left(\frac{t-\tau}{s}\right) \cdot ds \cdot d\tau \qquad (4)$$

Since the scale factor a is proportional to the inverse of the frequency ω, the value $(W_\psi f)(s_0,\tau_0)$ exhibits the frequency content of $f(t)$ in a frequency interval centered around $\omega_0 = s_0^{-1}$ at the time interval centered around $t = \tau_0$. The continous wavelet transform maps a signal of one independent variable t of two independent variables s,τ. Thus, from a computational point of view, this transform is not efficient. One way to solve this problem is to sample the continous wavelet transform on a two dimensoinal grid $(s_j, \tau_{j,k})$. If we choose the dyadic scales $s_j = 2^j$ and $\tau_{j,k} = k \cdot 2^j$ we obtain the dyadic discrete wavelet transform:

$$(W_\psi f)(2^j, k2^j) = < f(t), \psi_{j,k}(t) > \qquad (5)$$

where $< \bullet,\bullet >$ denotes the inner product in $\mathbf{L}_2(\mathbf{R})$. The dyadic sampling is a very natural choice for computers.We can construct functions [11]:

$$\left\{\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}} \psi\left(\frac{t-k\cdot 2^j}{2^j}\right)\right\}_{(j,k)\in \mathbf{Z}^2} \qquad (6)$$

to form an ortonormal basis for signal representations.

Multiresolution theory based on multiresolution analysis and other mathematical tools gives a foundation for exact description of expansion on orthogonal bases in a Hilbert space $\mathbf{L}_2(\mathbf{R})$. A multiresolution analysis consists of a sequence of embedded closed subspaces $\cdots \mathbf{V}_2 \subset \mathbf{V}_1 \subset \mathbf{V}_0 \subset \mathbf{V}_{-1} \subset \mathbf{V}_{-2} \cdots$ such that they posses the following properties: upward completeness, downward completeness, scale invariance, shift invariance, and existence of an orthonormal basis $\{\varphi(t-n) | n \in \mathbf{Z}\}$, for $\mathbf{V}_0$, where $\varphi \in \mathbf{V}_0$. The function $\varphi(t)$ is called the scaling function. The scaling function composes an orthonormal basis $\{\varphi_{m,n}\}$, $n \in \mathbf{Z}$ for the space $\mathbf{V}_m$

$$\varphi_{m,n}(t) = 2^{-m/2} \varphi(2^{-m} t - n), \text{ for } m,n \in \mathbf{Z} \qquad (7)$$

also exists an orthonormal basis [3]

$$\psi_{m,n}(t) = 2^{-m/2}\psi(2^{-m}t - n), \text{ for } m,n \in \mathbf{Z} \quad (8)$$

such that $\{\psi_{m,n}\}$, $n \in \mathbf{Z}$ is an orthonormal basis for a space $\mathbf{W}_m$, where $\mathbf{W}_m$ is the orthogonal complement of the space $\mathbf{V}_m$ in $\mathbf{V}_{m-1}$. The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is broken down into many lower resolution components [6]. This is called the wavelet decomposition tree.

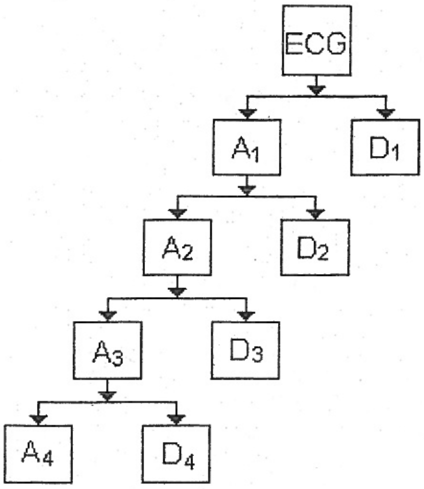

fig.1 Wavelet decomposition tree

II. THE MAIN ECG SIGNAL PARAMETERS

The electrocardiogram, or the ECG, is a time-varying signal that measures the electrical activity (on the surface of the human body) of the heart. Each heartbeat is a complex of distinct cardiological events, represented by distinct features in the ECG waveform. These features represent either depolarization (electrical discharging) or repolarization (electrical recharging) of the muscle cells in particular regions of the heart. Figure 2 shows a (human) ECG waveform and the associated parameters (features)

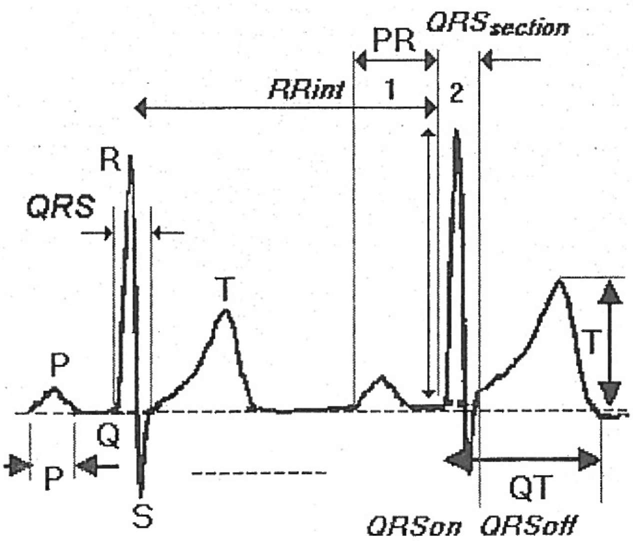

fig 2. The most important parameters of an ECG signal

The standard parameters of the ECG waveform are the P wave, the QRS complex and the T wave. Additionally a small U wave (with an uncertain origin) is ocassionally present. The cardiac cycle begins with the P wave, which corresponds to the period of atrial depolarization in the heart. This is followed by the QRS complex, which is usually the most relevant (recognizable) feature of an ECG waveform. The QRS complex corresponds to the period of ventricular depolarization. The start and end points of the QRS complex are referred to as Q and J points.

The T wave follows the QRS complex and corresponds to the period of ventricular repolarization. The end point of the T wave represents the end of the cardiac cycle (presuming the absence of U wave). The durations (time between the onset and offset) of particular parameters of the ECG (referred as an time interval) is of great importance since it provides a measure of the state of the heart and can show the presence of cerTain cardiological conditions. In practice, interval measurements, wave interpretations are carried out manually by ECG specialists

III. METHODOLOGY

The recognition of almost all ECG parameters is based on a fixed point identifiable at each cycle. R-peak is suitable for use as the datum point, because it has the largest amplitude and sharpest waveform that can be extracted from ECG. The time and amplitude measurements can be performed when the apex of the R-peak is detected at each cycle.

As the digitized ECG is a finite signal, its boundaries are usually abrupt. These abrupt cuts of the signal make it discontinuous. This introduces a smearing (decrease and spread) of all the estimated frequency peaks. In order to avoid this, the calculation of the FFT is applied to the windowed ECG. The windowing aims to smoothly decrease the boundary of the ECG signal to zero, removing its discontinuity. The limitation of this approach is that windowing reduces the frequency resolution and therefore lowers the quality of the estimation of the ECG signal frequencies. Another unavoidable limitation of the Fourier transformation for the ECG analysis is that this technique does not provide insight into exact location of frequency components in time. The frequency content of the ECG varies in time; the QRS complex is a high frequency wave whereas the T wave contains low-frequency components. Therefore, the need for an accurate description of the ECG frequency contents according to their location in time is essential. Utilization of time-frequency representation in quantitative electrocardiology is thus justified. The wavelet transformation is based on a set of analyzing wavelets allowing the decomposition of the ECG signal in a set of coefficients. Each analyzing wavelet has its own time duration, time location and frequency band. The wavelet transform provides a description of the signal in the time-scale domain allowing the representation of the temporal features at different resolutions, according to their frequency content. Noises and artefacts can be avoided considering their different contribution at various scales.

The wavelet coefficient resulting from the wavelet transformation corresponds to a measurement of the ECG components in this time segment and frequency band.

Usually an ECG signal analysis assume the existence of these steps: signal preprocessing (filtering of base line wandering, power lines interferences, signal denoising), features extraction, parameter estimation. In all of these, wavelet analysis can be applied with good results.

In all applications were used signals from MIT-BIH database with annotations from specialists (cardiologists), all methods were developed under Matlab (and Wavelet Toolbox). The main idea of wavelet analysis is to find a function (a basis function) which properties are appropiate to the analysed signal, to obtain maximum of information with less coefficients. The next figure presents how the multiresolution analysis based feature extraction is carried out

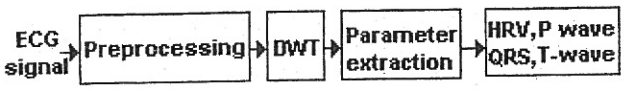

figure 3 Multiresolution based feature extraction

The ECG signal is noisy. This is caused by dynamic nature of the signal source and the dynamic systems between the source and recording electrodes. At first, the signals from database were filtered, denoised and after baseline wandering was removed, using wavelet approximation. The basic denoising concept is to decompose the signal at different scales (the largest quantity of information about noise is usually contained by the first scales detail coefficients)[4].

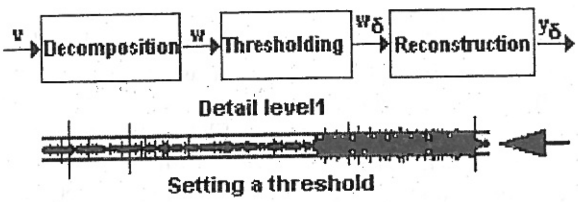

fig 3 Wavelet based denoising [5]

Wavelet decomposition (multiresolution analysis in fact) is useful in revealing signal trends, a goal that is complementary to the one of revealing a signal hidden in another signal. Usually the trend is the slowest part of the signal, the long-term evolution parameter. If the signal itself contains slow changes, then succesive approximations (average components) can reveal them.

The drift of the baseline due to the respiration can be represented as a sinusoidal component at the frequency of respiration added to the ECG signal. Typical parameters are: amplitude variation -15 percent of peak to peak ECG amplitude, frequency: 0.15 to 0.3 Hz

For baseline-wandering removal, at first, was identified the main low frequency component from the (already filtered) ECG signal, using the properties of wavelet decomposition.

For this, was used an 8^{th} level decomposition (using biorthogonal wavelets), the main low frequency component was identified as a weighted sum of the 6^{th} , 7^{th} and 8^{th} approximation. The weights were determined empirically, these are strongly correlated with the type of wavelet functions which were used Results are presented on next figure.

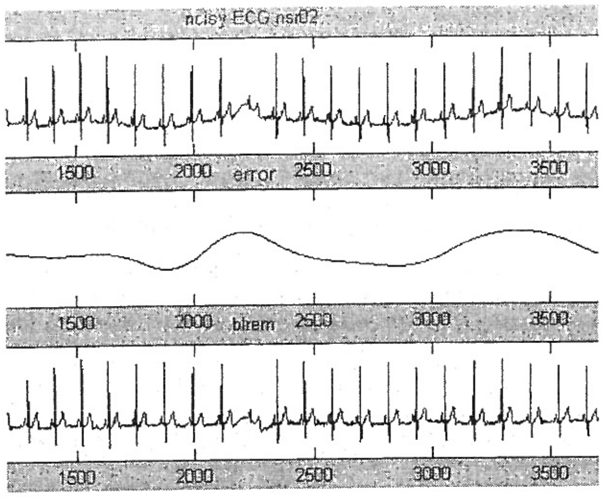

fig 4. Baseline wandering removal from ECG signals

The parameters of the ECG signal are obtained by the wavelet decomposition dyadic tree (DWT).To estimate the said ECG parameters in this work the following algorithm was used:

1.Select a wavelet-transformed ECG block data
2.Determination of the R wave location (as maxima) (first scale) (which occurs at the zero crossing point between the most proeminent maxima). Additionally, in this step the maxima can be identified as zero crossing points of the first derivative
3.Determination of R-R intervals, as R-R distances
4.Determination of Q, S points as the first zero crossing point before and after R wave
5.Elimination of the QRS from the signal to obtain the other parameters
6.Determination of the P wave location (as maxima) (scales3,4) (the same procedure as in 2), and the P-Q distance
7.Elimination of the P wave from the signal (same as 5)
8.Determination of the T wave location (as the remained maxima) (scales 3,4) and S-T segments durations

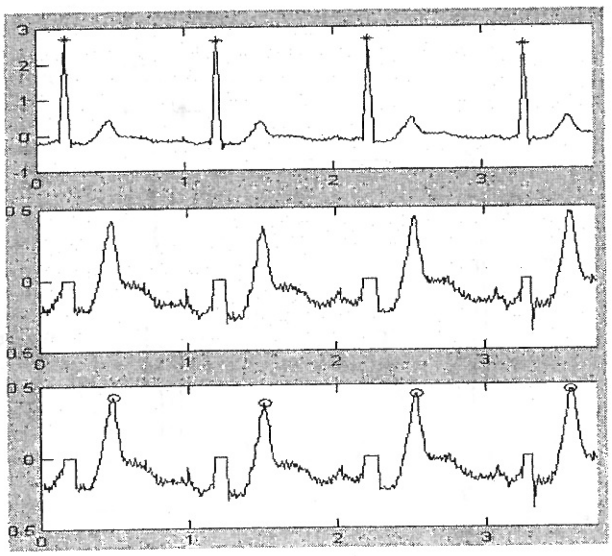

fig 5. R wave detection, QRS complex removal, T wave identification

The removal of already identified waves is made by replacing with zeros the components between the onset and offset moments (identified at steps 3 and 6). This procedure is presented on next figure.

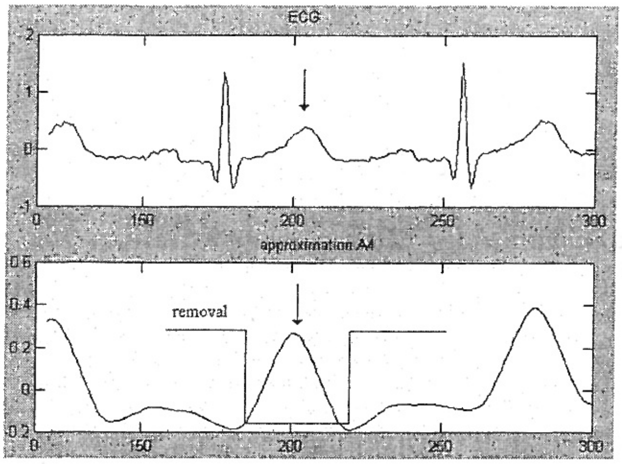

figure 6. Removal of a part of the signal

This procedure allows also the determination of the areas of significant waves in order to identify important modifications (as QRS complex, P wave, T wave areas) in ECG signal

IV. RESULTS

This algorithm, developed for single lead ECG signals, leads to determine the main parameters as R to R duration, P and T waves localizations, Q-T, P-Q durations. Were used over 27 files from the MIT-BIH database, signals containing normal sinus rhythms and signals with abnormalities in order to find the main parameters.

V. CONCLUSIONS

The results obtained (processing mainly ECG signals from normal sinus database) were compared with annotated files from ECG databases, and gave very promising results: R wave detection around 92%, R-R interval (HRV) determination 90% QRS complex detection over 90%, T wave detection or localization 81%, P wave detection or localization 78%. (over 33 files used from MIT-BIH database, signals were denoised and with the baseline drift removed). The present study is based on biorthogonal wavelets. It has been shown [5] that these wavelets are ideally suited for the purpose since they excite the various morphologies of the ECG signals at different scales. The wavelet transform has proven to be a good tool for ECG signal analysis. It achieves a sufficiently high level of reliability. It enables to detect required values of selected attributes in just a few steps which is important from the point of view of time required for processing. As a further work, an artificial neuronal network will be implemented (trained with a set of normal sinus beats) for ECG events analysis and abnormalities detection, where the extracted values (parameters) will be used as input values for a classification system

VI. REFERENCES

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