Three Degrees of Freedom Parallel Structure Used for the TV Satellite Dish Orientation

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Abstract **-In the paper are presented studies regarding the possibility to use a three degrees of freedom parallel mechanism for the TV satellite antenna orientation. The advantage to use a spatial parallel mechanism consists of their good stiffness and their capability to manipulate heavy loads. The relationships between the satellite angles (Azimuth and Elevation) and the Euler angles are derived. The algorithms for the geometric and kinematic models are solved. Based on these algorithms, the actuated joint coordinates and the passive joint coordinates are computed, corresponding to the antenna orientation over two geostationary satellites. Using a numerical simulation, the diagrams for the actuated joint coordinates and speeds are represented.**

I. INTRODUCTION

The TV reception via satellites is a relative new technique. In order to ensure an almost permanent connection Earth-satellite and reverse, it was necessary to use geostationary satellites, which must have the orbital plane in the equator plane of the Earth, and the height with respect to the ground at about 36000 Km. The first geostationary satellites were launched in USA between 1963-1964 [1]. The TV public broadcasting with help of geostationary satellites has begun in 1978.

On a geostationary satellite act simultaneously the gravitation force and the centrifugal force. In order to ensure the satellite stability, these two forces must be equal. This condition together with the phasing condition yields to the values of the orbital parameters of the geostationary satellite:

- -satellite rotation period $T=86164$ *sec.*;
- -satellite height $h=36779$ *Km*;
- -satellite orbit radius $d=r+h=42156$ *Km*, r=6377 Km represents the earth radius on the equator;
- -satellite speed $v_s=3.075$ *Km/s*.

Because the solid angle of the antenna is only 3^0-5^0, the antenna orientation could be made only knowing at first the satellite position with respect to the receiver. Generally, the S satellite and the R receivers are located on different meridians (figure 1).

The receiver position is given by two coordinates: θ_R - receiver latitude and λ_R - receiver longitude. The satellite position is given by: h – satellite altitude and λ_S - satellite longitude.

The parameters, which determine the relative position of the satellite with respect to the receiver, are:

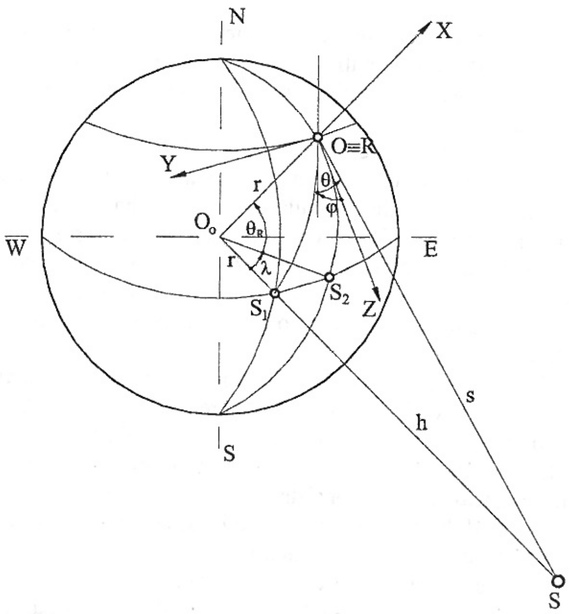


Fig. 1. The relative position of the satellite with respect to the receiver

θ - the elevation angle means the angle between the RS direction and the tangent plane in R on the Earth (the angle between RS and the tangent on the RS_1 circle);

φ - Azimuth angle means the angle between the receiver meridian and the tangent on the RS_1 circle;

s – the distance between the satellite and the receiver.

These three parameters could be computed by mean of the known relationships from the spherics:

$$\varphi = a\,tan\frac{tan(\lambda_S - \lambda_R)}{sin\theta_R} \quad (1)$$

$$\theta = a\,tan\frac{cos\,\delta - \frac{r}{d}}{sin\,\delta};\quad \delta = a\,cos[cos\,\theta_R\ cos(\lambda_S - \lambda_R)] \quad (2)$$

$$s = h\sqrt{1 + 2\frac{r}{h}(1 - cos\,\delta)} \quad (3)$$

Practically, for the antenna orientation it is important to find out only the angles φ and θ.

The low weight receiver antenna orientation could be made with an open kinematic chain mechanism of type RR (figure 2). Another solution could be a mechanism with closed kinematic chain of type RT (figure 3).

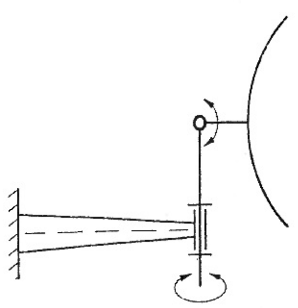


Fig. 2 Serial kinematic chain

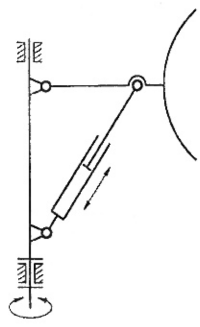


Fig. 3. Closed kinematic chain

The heavy antennas are located in zones with strong winds require orientation mechanisms with high rigidity. In this case a solution is the using of the parallel mechanisms.

Dunlop a.o. [2] presents a Stewart platform prototype for a radio antenna orientation that assures a wide operating range avoiding the singularities. They have found an original technological solution of equivalence for the upper joints in order to allow significant rotations for the mobile plate. The rotary actuators are mounted on the base and connected through Cardan joints to the screws, which vary the arms lengths.

Meschini a.o. [5] present the study of a parallel mechanism with six degrees of freedom of type MSSM (Minimal Simplified Symmetric Manipulator- MSSM) after [4] for a satellite antenna with double reflector. The mechanical system consists of two platforms which are connected through six extensible legs. On each platform is mounted a reflector for signaling.

In this paper, for the TV receiver antenna a parallel mechanism of type 3-RRS is proposed to be used. This mechanism has two orientation degrees of freedom and one translation degree of freedom. The workspace and the singularities for this mechanisms type was studied in [6]. The kinematic analysis of a similar mechanism of type 3-RTS was presented in [4].

The kinematic scheme of the 3-RRS mechanism is presented in the figure 4. It consists of: i) upper platform on which the antenna dish is mounted, ii)) three kinematic chains with an actuated rotation joint, a passive rotation joint and a spherical joint and iii) the base. The spherical joints centers are equally located on 120^0 and on the r distance with respect to the center of the mobile plate. The actuated joints centers are also equally located on 120^0 and on the R distance with respect to the center of the fixed plate. Theoretically, the passive spherical joint centers located on the mobile plate are imposed to remain each of them on a mobile curve with one degree of freedom with respect to the base (figure 5).

In the paper we propose the possibility to use the 3-RRS mechanism for the receiver TV antenna orientation. In this case, the geometric model is solved, deriving the connection between the relative coordinates of the satellite, the Cartesian coordinates of the mobile plate and the joint coordinates of the mechanism. The kinematic model is also established.

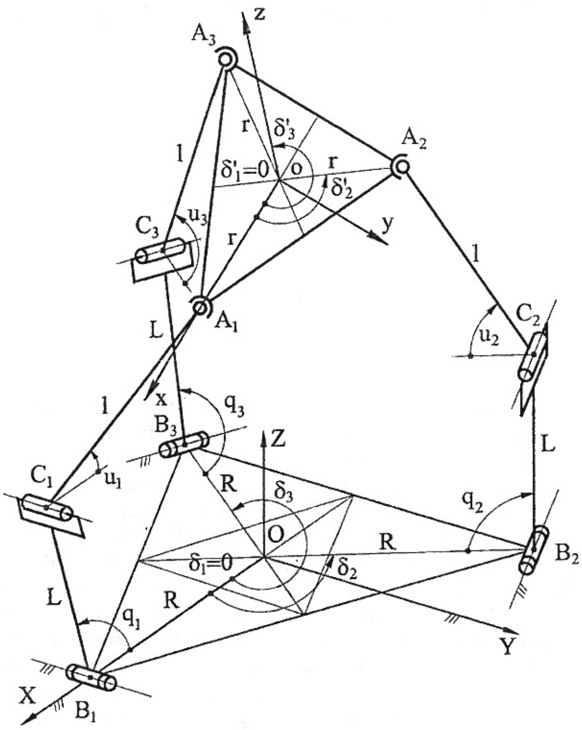


Fig. 4. The 3-RRS parallel mechanism

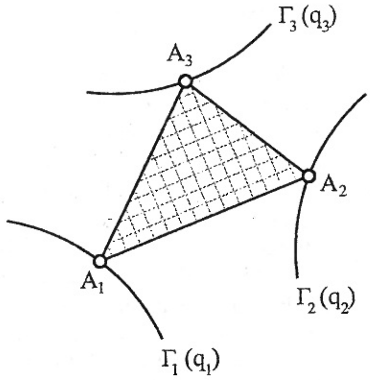


Fig. 5. Theoretical model

II. DERIVING OF THE GEOMETRIC CONTROL EQUATIONS

Knowing the position of the mobile plate $A_1A_2A_3$ (figure 4) with respect to the base $B_1B_2B_3$ through the Z coordinate of the platform centre and the Euler angles α, β, the joint coordinates q_i and the passive coordinates u_i are derived. Because the mechanism has only three degrees of freedom, the other coordinates X and Y of the mobile plate centre and the thirds Euler angle γ will be derived with respect to the Z, α, β parameters.

In order to establish the geometric control equations, the parametric equations of the guiding mobile curves are used, on which the centers of the spherical joints are moving with respect to the base:

$$\begin{cases} X_i = (R - Lcq_i - lcu_i)c\delta_i \\ Y_i = (R - Lcq_i - lcu_i)s\delta_i \qquad i = 1,2,3 \\ Z_i = Lsq_i + lsu_i \end{cases} \tag{4}$$

The Cartesian equations of these curves will be:

$$\begin{cases} (X_i c\delta_i + Y_i s\delta_i - R + Lcq_i)^2 + (Z_i - Lsq_i)^2 = l^2 & a) \\ X_i s\delta_i = Y_i c\delta_i & b) \end{cases} \tag{5}$$

In the paper the notations are: s=sin and c=cos.

On the other side, the absolute coordinates of the guided points A_i depend on the Cartesian coordinates of the mobile platform:

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \cdot \begin{bmatrix} rc\delta_i' \\ rs\delta_i' \\ 0 \end{bmatrix}; i = 1,2,3 \tag{6}$$

The rotation matrix has the following form:

$$[R_E]_{x(\alpha),y(\beta),x(\gamma)} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} = \tag{7}$$

$$= \begin{bmatrix} c\beta & s\beta s\gamma & s\beta c\gamma \\ s\alpha s\beta & c\alpha c\gamma - s\alpha c\beta s\gamma & -c\alpha s\gamma - s\alpha c\beta c\gamma \\ -c\alpha s\beta & s\alpha c\gamma + c\alpha c\beta s\gamma & -s\alpha s\gamma + c\alpha c\beta c\gamma \end{bmatrix}$$

By replacing in (5b) the expressions (6) and the values of the angles $\delta_i = \delta_i' = (i-1)\frac{2\pi}{3}, i = 1,2,3$, the following relations are derived:

$$X = \frac{1}{2} r(\alpha_1 - \beta_2) \tag{8}$$

$$Y = -r\beta_1 \tag{9}$$

$$\alpha_2 = \beta_1 \tag{10}$$

The condition $\alpha_2 = \beta_1$ leads to the equation $s\beta s\gamma = s\beta s\alpha$ which has the solution:

$$\gamma = \alpha \tag{11}$$

The final form of $[R_E]$ rotation matrix will be:

$$[R_E] = \begin{bmatrix} c\beta & s\alpha s\beta & c\alpha s\beta \\ s\alpha s\beta & 1 - (1 + c\beta)s^2\alpha & -\frac{1}{2}(1 + c\beta)s^2\alpha \\ -c\alpha s\beta & \frac{1}{2}(1 + c\beta)s^2\alpha & -1 + (1 + c\beta)c^2\alpha \end{bmatrix} \tag{12}$$

We consider the satellite orientation is located with the base on the vertical wall to the south, as in the figure 6.

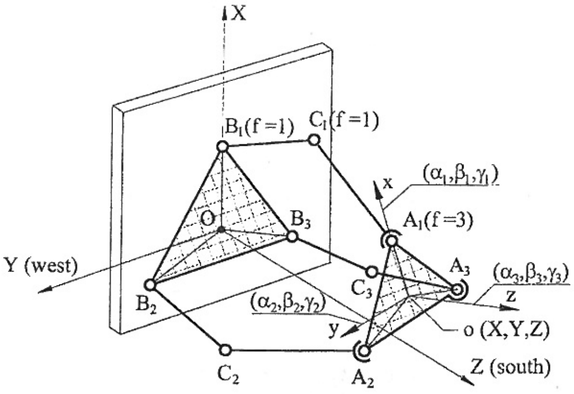


Fig. 6. Mechanism positioning

A geostationary satellite could be located only through two angles named azimuth (φ) and elevation (θ) (figure 1). The rotation matrix corresponding to the satellite angles has the form:

$$[R_S] = \begin{bmatrix} c\theta & 0 & s\theta \\ s\varphi s\theta & c\varphi & -s\varphi c\theta \\ -c\varphi s\theta & s\varphi & c\varphi c\theta \end{bmatrix} \tag{13}$$

From the condition that oz axis should be on the OS direction of the satellite:

$$\begin{aligned} & c\alpha s\beta = s\theta \\ & -\frac{1}{2}(1 + c\beta)s^2\alpha = -s\varphi c\theta \\ & -1 + (1 + c\beta)c^2\alpha = c\varphi c\theta \end{aligned} \tag{14}$$

the Euler angles are derived with respect to the satellite angles:

$$\alpha = a\tan\frac{s\varphi c\theta}{1 + c\varphi c\theta} \qquad \beta = a\tan 2\frac{\frac{s\theta}{c\alpha}}{\frac{s\alpha c\theta}{s\alpha c\alpha} - 1} \tag{15}$$

Thus the Cartesian coordinates X, Y, γ of the mobile plate has the expressions:

$$X = \frac{1}{2} r(c\beta + (1 + c\chi)s^2\alpha - 1) \tag{16}$$

$$Y = -r s\alpha s\beta \tag{17}$$

$$\gamma = \alpha \tag{18}$$

Now with the relations (6) the absolute coordinates of the guiding points could be derived.

To obtain the joint coordinates the equations (5a) are put into the form:

$$a_i cq_i + b_i sq_i = c_i \; ; i = 1,2,3 \tag{19}$$

where:

$$\begin{cases} a_i = 2L(X_i c\delta_i + Y_i s\delta_i - R) \\ b_i = -2LZ_i \qquad\qquad i = 1,2,3 \\ c_i = l^2 - L^2 - (X_i c\delta_i + Y_i s\delta_i - R)^2 \end{cases} \tag{20}$$

The solution of the equation system (19) is:

$$(sq_i)_{1,2} = \frac{b_i c_i \mp a_i\sqrt{a_i^2 + b_i^2 - c_i^2}}{a_i^2 + b_i^2} \qquad i=1,2,3$$

$$(cq_i)_{1,2} = \frac{a_i c_i \pm b_i\sqrt{a_i^2 + b_i^2 - c_i^2}}{a_i^2 + b_i^2}$$

$$(q_i)_{1,2} = a\tan 2\frac{(sq_i)_{1,2}}{(cq_i)_{1,2}} \tag{21}$$

Practically, there is possible only one of the solutions (21), depending on the initial mounting of the mechanism (figure 7).

From (1), the rotation angles form the passive joint could be computed:

$$u_i = a\tan 2\frac{Z_i - Lsq_i}{R - Lcq_i - X_i c\delta_i - Y_i s\delta_i} \qquad i = 1,2,3 \tag{22}$$

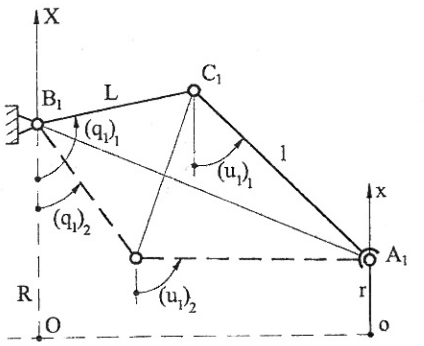


Fig. 7. Two solutions for the inverse geometric model

III. INVERSE KINEMATICS OF THE MECHANISM

Knowing the moving equations of the mobile plate:

$$\alpha = \alpha(t),\ \beta = \beta(t),\ Z = Z(t) \tag{23}$$

and the first order derivatives $\dot\alpha, \dot\beta, \dot Z$ and the second order derivatives $\ddot\alpha, \ddot\beta, \ddot Z$ with respect to the time, the joint and the passive speeds $\dot q_i, \dot u_i$ and the joint and the passive accelerations $\ddot q_i, \ddot u_i$ are derived.

Between the unit vectors of the mobile reference systems, those were obtained through the consecutive rotations with the angles α, β, α (figure 8), could be written the following relations:

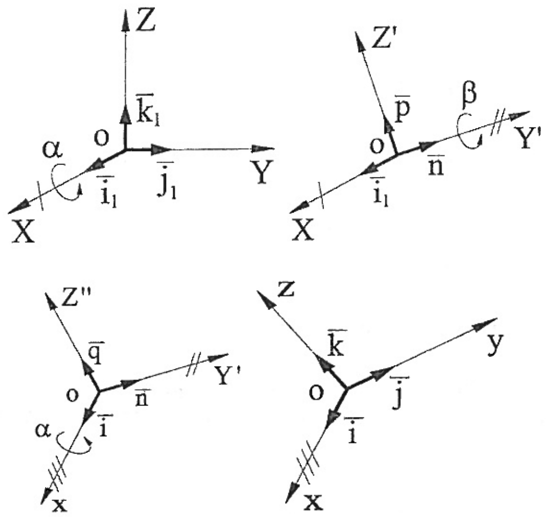


Fig. 8 Axis rotations

$$\begin{bmatrix} \bar i_1 \\ \bar j_1 \\ \bar k_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix} \cdot \begin{bmatrix} \bar i_1 \\ \bar n \\ \bar p \end{bmatrix}; \quad \begin{bmatrix} \bar i_1 \\ \bar n \\ \bar p \end{bmatrix} = \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \cdot \begin{bmatrix} \bar i \\ \bar n \\ \bar q \end{bmatrix};$$

$$\begin{bmatrix} \bar i \\ \bar n \\ \bar q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix} \cdot \begin{bmatrix} \bar i \\ \bar j \\ \bar k \end{bmatrix} \tag{24}$$

The analytic expression of the angular speed is:

$$\bar\omega = \dot\alpha \bar i_1 + \dot\beta \bar n + \dot\alpha \bar i \tag{25}$$

The scalar components of the angular speed on the oxyz reference system axes of the mobile plate are derived by multiplying the relation (27) successively through the unit vectors $\bar i, \bar j, \bar k$.

$$\begin{cases} \omega_x = \dot\alpha(1 + c\beta) \\ \omega_y = \dot\alpha\, s\alpha\, s\beta + \dot\beta\, c\alpha \\ \omega_z = \dot\alpha\, c\alpha\, s\beta - \dot\beta\, s\alpha \end{cases} \tag{26}$$

The modulus of the angular speed is:

$$\omega = \sqrt{\dot\beta^2 + 2\dot\alpha^2(1 + c\beta)} \tag{27}$$

The the angular speed projections on the base axes are:

$$\begin{cases} \dot X = \frac{1}{2} r[\dot\alpha(1 + c\beta)s2\alpha - \dot\beta(1 + s^2\alpha)s\beta \\ \dot Y = -r(\dot\alpha\, c\alpha\, s\beta + \dot\beta\, s\alpha\, c\beta) \\ \dot Z = \dot Z \end{cases} \tag{28}$$

The scalar components of the A_i guiding point's speeds result from:

$$\begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{Z}_i \end{bmatrix} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} + [R_E] \cdot \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & \omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \cdot \begin{bmatrix} r c\delta_i' \\ r s\delta_i' \\ 0 \end{bmatrix}, \; i = 1,2,3 \tag{29}$$

Differentiating the equations

$$\begin{cases} X_i c\delta_i + Y_i s\delta_i = R - L cq_i - l cu_i \\ Z_i = L sq_i + l su_i \end{cases} \tag{30}$$

with respect to the time, the angular speeds are derived:

$$\begin{bmatrix} \dot{q}_i \\ \dot{u}_i \end{bmatrix} = \frac{1}{c\delta_i \, s(q_i - u_i)} \begin{bmatrix} \frac{cu_i}{L} & -\frac{su_i}{L} \\ -\frac{cq_i}{l} & \frac{sq_i}{l} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_i \\ \dot{Z}_i \end{bmatrix}, \; i = 1,2,3 \tag{31}$$

It was taken into account that

$$\dot{Y}_i c\delta_i = \dot{X}_i s\delta_i \tag{32}$$

The joint coordinates could be derived by differentiating the equations:

$$(X_i c\delta_i + Y_i s\delta_i - R + Lcq_i)^2 + (Z_i - Lsq_i)^2 = l^2, \; i = 1,2,3$$

It results:

$$\dot{q}_i = \frac{(X_i c\delta_i + Y_i s\delta_i - R + Lcq_i)(\dot{X}_i c\delta_i + \dot{Y}_i s\delta_i) + (Z_i - Lsq_i)\dot{Z}_i}{(X_i c\delta_i + Y_i s\delta_i - R)Lsq_i + Z_i Lcq_i} \tag{33}$$

Differentiating the equation (31), the relationship between accelerations is derived:

$$\begin{bmatrix} \ddot{q}_i \\ \ddot{u}_i \end{bmatrix} = \frac{1}{c\delta_i \, s(q_i - u_i)} \begin{bmatrix} \frac{cu_i}{L} & -\frac{su_i}{L} \\ -\frac{cq_i}{l} & \frac{sq_i}{l} \end{bmatrix} \cdot \begin{bmatrix} \ddot{X}_i \\ \ddot{Z}_i \end{bmatrix} + \frac{1}{c\delta_i \, s(q_i - u_i)} \begin{bmatrix} \frac{-\dot{u}_i su_i}{L} & -\frac{\dot{u}_i cu_i}{L} \\ \frac{\dot{q}_i sq_i}{l} & \frac{\dot{q}_i cq_i}{l} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_i \\ \dot{Z}_i \end{bmatrix} + \frac{-(\dot{q}_i - \dot{u}_i) c(q_i - u_i)}{c\delta_i \, s^2(q_i - u_i)} \begin{bmatrix} \frac{cu_i}{L} & -\frac{su_i}{L} \\ -\frac{cq_i}{l} & \frac{sq_i}{l} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_i \\ \dot{Z}_i \end{bmatrix}; \; i = 1,2,3 \tag{34}$$

where $\ddot{X}_i$ and $\ddot{Z}_i$ result from the relation known from the kinematics:

$$\begin{bmatrix} \ddot{X}_i \\ \ddot{Y}_i \\ \ddot{Z}_i \end{bmatrix} = \begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} + [R_E] \cdot \left(\begin{bmatrix} 0 & -\dot{\omega}_z & \dot{\omega}_z \\ \dot{\omega}_z & 0 & -\dot{\omega}_x \\ -\dot{\omega}_y & \dot{\omega}_x & 0 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \right) \cdot \begin{bmatrix} r c\delta_i' \\ r s\delta_i' \\ 0 \end{bmatrix} \tag{35}$$

in which:

$$\ddot{X} = \frac{1}{2} r[(\ddot{\alpha} s2\alpha + 2\dot{\alpha}^2 c2\alpha)(1 + c\beta) - (\ddot{\beta} s\beta + \dot{\beta}^2 c\beta)(1 + s^2\alpha) - 2\dot{\alpha}\dot{\beta} s\beta s2\alpha]$$

$$\ddot{Y} = -r[\ddot{\alpha} c\alpha s\beta + \ddot{\beta} s\alpha c\beta - (\dot{\alpha}^2 + \dot{\beta}^2) s\alpha s\beta + 2\dot{\alpha}\dot{\beta} c\alpha c\beta] \tag{36}$$

$$\begin{cases} \dot{\omega}_x = \ddot{\alpha}(1 + c\beta) - \dot{\alpha}\dot{\beta} s\beta \\ \dot{\omega}_y = \ddot{\alpha} s\alpha s\beta + \ddot{\beta} c\alpha + \dot{\alpha}^2 c\alpha s\beta + \dot{\alpha}\dot{\beta} s\alpha(c\beta - 1) \\ \dot{\omega}_z = \ddot{\alpha} c\alpha s\beta - \ddot{\beta} c\alpha - \dot{\alpha}^2 s\alpha s\beta + \dot{\alpha}\dot{\beta} c\alpha(s\beta - 1) \end{cases} \tag{37}$$

IV. NUMERICAL RESULTS

An antenna mechanism of type 3-RRS, located in Cluj-Napoca, Romania ($\lambda_R = 23.6^o, \theta_R = 46.8^o$) with the following geometric parameters:

$R = 0.45\, m, L = 0.4\, m, l = 0.60\, m, r = 0.15\, m,$

$\delta_1 = \delta_1' = 0, \delta_2 = \delta_2' = 120^o, \delta_3 = \delta_3' = 240^o$

and two satellites: a) INTELSAT 907 ($\lambda_S = 27.5^o\, VEST$) and b) INTELSAT ($\lambda_S = 62^o\, EST$).

Using the proposed algorithm for the geometric model with Z=0.6 m the following output data were derived:

a. for INTELSAT 907

- The satellite angles: $\varphi = -59.536^o$; $\theta = 17.148^o$;
- The Euler angles: $\alpha = -29.027^o$, $\beta = 19.706^o$;
- The coordinates of the mobile plate center: X=0.03 m, Y=0.025 m;
- The actuated joint coordinates: $q_1 = 131.312^o$; $q_2 = 127.061^o$; $q_2 = 102.208^o$;
- The passive joint coordinates $u_1 = 25.182^o$; $u_2 = 19.059^o$; $u_3 = 34.304^o$

a. for INTELSAT 902

- The satellite angles: $\varphi = 47.394^o$; $\theta = 24.534^o$;
- The Euler angles: $\alpha = 22.508^o$, $\beta = 26.71^o$;
- The coordinates of the mobile plate center: X=0.03 m, Y=-0.026 m;
- The actuated joint coordinates: $q_1 = 129.074^o$; $q_2 = 106.265^o$; $q_2 = 129.569^o$;
- The passive joint coordinates $u_1 = 22.249^o$; $u_2 = 33.841^o$; $u_3 = 23.143^o$

We consider for a 3-RRS parallel manipulator with the same geometric parametric the following displacement laws of the platform:

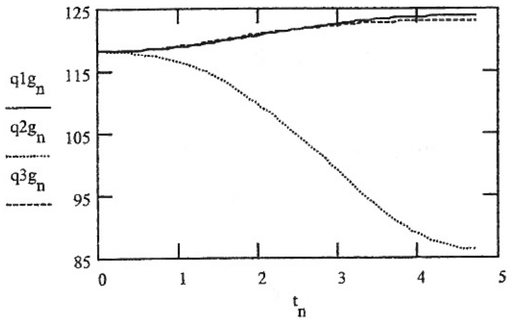
$$Z = 0;\; \alpha = \psi(t)/12;\; \beta = \psi(t)/12$$

where:

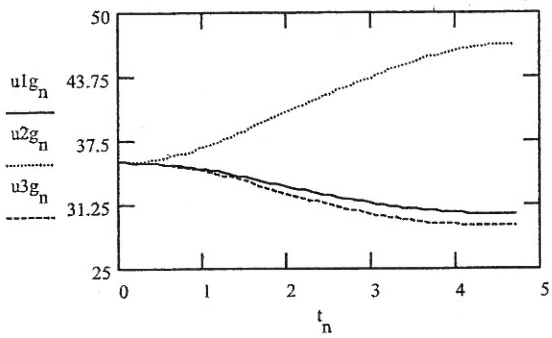
$$\psi(t)=\begin{cases}\dfrac{2}{\pi}\cdot t^2 & if\ 0\le t<\dfrac{\pi}{2}\\ 2\cdot\left(t-\dfrac{\pi}{2}\right)+\dfrac{\pi}{2} & if\ \dfrac{\pi}{2}\le t<\pi\\ -2\cdot\dfrac{(t-\pi)^2}{\pi}+2\cdot(t-\pi)+3\cdot\dfrac{\pi}{2} & if\ \pi<t\le\dfrac{3\pi}{2}\end{cases}$$

Using the algorithms for geometric and kinematic models the following diagrams with respect to the time were derived:

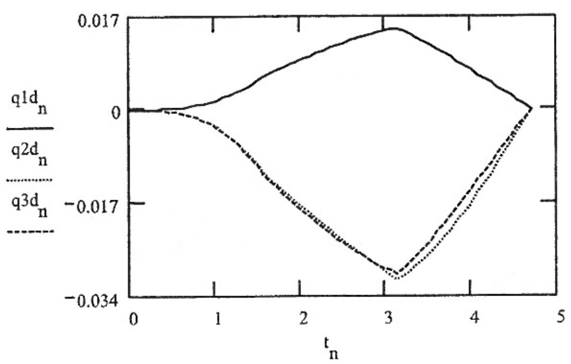
-The actuated joint coordinates



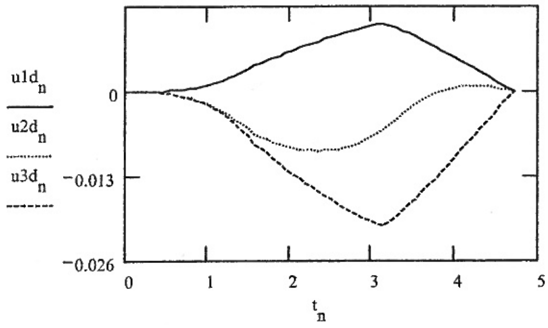
-The passive joint coordinates:



-The actuated joint velocities



-The passive joint velocities



CONCLUSIONS

With this type of parallel mechanism is possible the orientation of the dish TV antenna for "catching" the most used geostationary satellites. The using of a parallel mechanism offers the advantage of a high stiffness and the possibility the distance control of the dish with respect to the fixed base.

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