Low Velocity Friction Modelling with Application for DC Servo Control*

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Abstract – **This paper presents a novel friction model developed for low velocity regimes. The model can easily be incorporated in adaptive control laws because it is linear in parameters. Adaptive friction compensation method was developed for DC servo motors using the introduced model. The stability and performances of the control scheme with the proposed control law was discussed using Lyapunov techniques. Simulations also were performed to show the performances of the applied control algorithm.**

I. INTRODUCTION

Friction is universally present in the motion of bodies in contact. It plays a role in the simplest actions of living, such as walking, grasping and stacking. In servo controlled machines friction has an impact in all regimes of operation. In high precision positioning systems it is inevitable to know the value of the friction force to assure good control characteristics and to avoid some undesired effects such as limit cycle and steady state error.

Many models were developed to explain the friction phenomenon [1, 2]. The introduced models are based on experimental results rather than analytical deductions.

Kinetic Friction Model: The classical friction model were developed toward by Coulomb who discovered that the friction force depends on the sign of the velocity. So the friction force can be written in the following form:

$$F_f = F_C sign(v) \tag{1}$$

where F_C is a constant and v denotes the relative velocity between the two surfaces in contact.

Kinetic + Static Friction Model: The static friction, introduced initially by Artur Morin, represents the force necessary to initiate motion from rest and in most of the cases its value is grater than the kinetic friction. This effect was introduced in the kinetic friction model:

$$F_f = F_S\eta(v) + F_C sign(v),\ \eta(v) = \begin{cases} 1 & if\ v=0 \\ 0 & otherwise \end{cases} \tag{2}$$

where F_S is a constant.

Kinetic + Static + Viscous Friction Model: The viscous term is the friction component that is proportional to velocity. This term has a dominant influence when the contact of the bodies in motion are lubricated with oil or grace (hydrodynamic lubrication). It was introduced by Reynolds who studied the friction occurring in fluids. The static plus kinetic plus viscous friction model nowadays is the most commonly used in engineering:

$$F_f = F_S\eta(v) + F_C sign(v) + F_V v \tag{3}$$

where F_V is a constant.

Striebeck friction model: Many servo-controlled machines are lubricated with oil or grace (hydrodynamic lubrication). Tribological experiments showed that in the case of lubricated contacts the simple static +kinetic + viscus model cannot explain some phenomena in low velocity regime, such as the Striebeck effect. This friction phenomenon arises from the use of fluid lubrication and gives rise to decreasing friction with increasing velocities.

To describe this low velocity friction phenomenon four regimes of lubrications are treated. *Static Friction:* the junctions deform elastically and there is no excursion until the control force do not reach the level of static friction force. *Boundary Lubrication:* this is also solid to solid contact, the lubrication film is not yet built. The velocity is not adequate to build a solid film between the surfaces. A sliding of friction force occurs in this domain of low velocities. The friction force decreases with the increasing of velocity but generally is assumed that friction in boundary lubrication is higher than for fluid lubrication (regimes three and four). *Partial Fluid Lubrication:* the lubricant is drawn into the contact area through motion, either by sliding or rolling. The greater viscosity or motion velocity, the thicker the fluid film will be. Until the fluid film is not thicker than the height of aspirates in the contact regime, some solid-to-solid contacts will also influence the motion. *Full Fluid Lubrication:* When the lubricant film is sufficiently thick, separation is complete and the load is fully supported by fluids. The viscous term dominates the friction phenomenon, the solid-to-solid

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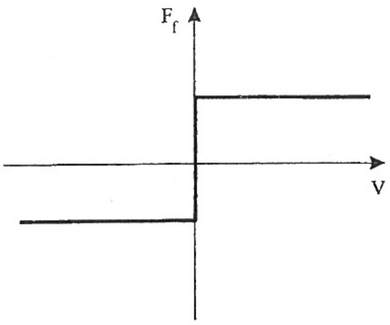

Fig. 1: Kinetic Friction

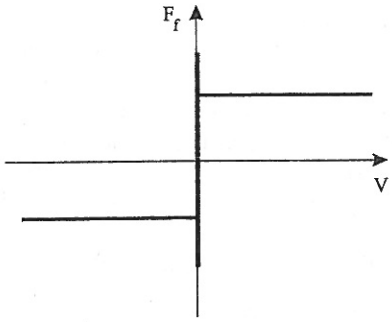

Fig. 2: Kinetic+Static Friction

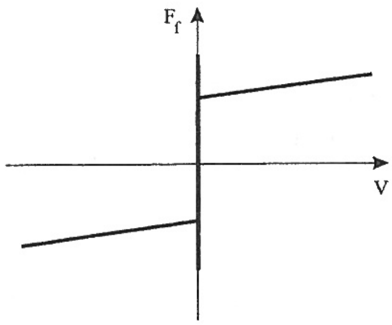

Fig. 3: Kinetic+Static+Viscous Friction

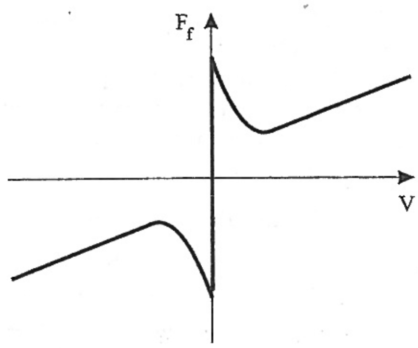

Fig. 4: Striebeck Friction (low velocities)

contact is eliminated and the friction is 'well behaved'. The value of the friction force can be considered as proportional with the velocity.

From these domains results a highly nonlinear behavior of the friction force (see Fig. 4). Near zero velocities the friction force decreases in function of velocity and at higher velocities the viscous term will be dominant so the friction force increases with velocity. Moreover the friction also depends on the sign of velocity with an abrupt change when the velocity pass through zero.

For the moment no predictive model of the Striebeck effect is available. Several empirical models were introduced to explain the Striebeck phenomena: Tustin model, exponential in velocity $e^{-|v|/v_S}$, Gaussian model $e^{-(v/v_S)^2}$, Lorentzian model $1/(1+(v/v_S)^2)$. The constant value v_S is the Striebcek velocity which describes the shape of the Striebeck curve. This terms can be introduced in the previously presented model (3) to obtain a more precise friction model. Note that the static friction term in the form as in the relations (2) and (3) is not used because the Striebeck term describes more precisely the friction phenomena near zero velocities.

$$\begin{aligned} & Tustin\ model: \\ F_f &= (F_C + (F_S - F_C)e^{-|v|/v_S})sign(v) + F_V v \quad (4) \\ & Gaussian\ model: \\ F_f &= (F_C + (F_S - F_C)e^{-(v/v_S)^2})sign(v) + F_V v \quad (5) \\ & Lorentzian\ model: \\ F_f &= (F_C + (F_S - F_C)1/(1+(v/v_{sw})^2))sign(v) + \\ & F_V v \quad (6) \end{aligned}$$

Remark: Note that at zero velocities the value of friction force can not be greater than the tangential force which acts on the junction. Until the tracking force does not reach the level of the static friction (F_S) the value of the friction force is equal with the value of the tangential tracking force. This effect can be introduced in Tustin friction model as follows:

$$F_f = \begin{cases} \tau, & if\ \tau < F_S\ and\ v = 0 \\ (F_C + (F_S - F_C)e^{-|v|/v_S})sign(v) + F_V v, & otherwise \end{cases} \quad (7)$$

where τ denotes the tangential control force.

II. LINEARLY PARAMETERIZED STATIC FRICTION MODELS

The parameters of the friction model may change as a function of normal forces in contact, temperature variations, humidity, lubricant conditions, material proprieties, dwell time and other factors that can hardly be controlled. (The *dwell time* represents the time interval a junction spends in the stuck when the machine is not moving. It effects mostly the static friction term.) This is why the parameters of the friction models should be considered as time varying and for precise positioning is not enough an a-priori determination of them. Obviously the friction parameter variation is a slow process but it could affect the performances of the machine over time. This is why the on-line estimation methods are so popular for friction compensation.

To apply the well known adaptive control schemes for friction compensation it is desirable that the friction model could be written in a linearly parameterized form, namely as a scalar product between a known regressor vector $\underline{\xi}_F$ and an unknown parameter vector $\underline{\theta}_F$ ($F_f = \underline{\theta}_F^T \underline{\xi}_F$).

In the other hand the friction parameters could change even in the function of sign of velocity. This is why it is recommended to use different friction parameters in the positive and negative velocity regimes.

The previously introduced friction models should be rewritten or modified in such way to obtain the previously presented requirements (linear parameterization, different parameters sets for positive and negative velocities) and in the same time to keep the qualitative characteristics of the original models. To achieve these, first of all let us introduce the following switching function:

$$\mu(v) = \begin{cases} 1 & if\ v \geq 0 \\ 0 & otherwise \end{cases} \tag{8}$$

Note that the function μ has the propriety: $\mu(-v) = 1 - \mu(v)$.

Kinetic and Kinetic+Viscous Model: These simple models could be easily rewritten if we use the switching function μ defined in (8) and we consider different parameters for pôsitive and negative velocities.

Kinetic friction:

$$\begin{aligned} F_f &= \underline{\theta}_F^T \underline{\xi}_F(v) \qquad (9) \\ \underline{\theta}_F &= (F_{C+}\ F_{C-})^T \quad \underline{\xi}_F(v) = (\mu(v)\ \mu(-v))^T \end{aligned}$$

Kinetic + Viscous friction:

$$\begin{aligned} F_f &= \underline{\theta}_F^T \underline{\xi}_F(v) \qquad (10) \\ \underline{\theta}_F &= (F_{C+}\ F_{V+}\ F_{C-}\ F_{V-})^T \\ \underline{\xi}_F(v) &= (\mu(v)\ \mu(v)v\ \mu(-v)\ \mu(-v)v)^T \end{aligned}$$

The subindex $'+'$ means that the parameter is used only in positive velocity regime and the subindex $'-'$ means that the respective parameter is applied only for negative velocities. Note that the static friction cannot be incorporated in this models and it cannot be estimated on line because it occurs only at zero velocities. This is why we need a more accurate model near zero velocities.

A. *Linearized friction model for low velocities*

The model, introduced in this paper, was developed based on the Tustin model (4). For the simplicity only the positive velocity domain is considered, but same study can be made for the negative velocities. Let us consider that our mechanical system moves in $0 \dots v_{max}$ velocity domain.

Let us consider a linear approximation for the exponential curve represented by two lines: d_{1+} which cross through the $(0, F_f(0))$ point and it is tangent to curve and d_{2+} which passes through the $(v_{max}, F_f(v_{max})$ point and tangential to curve. These two lines meet each other at the v_{sw} velocity. In the domain $0 \dots v_{sw}$ the d_{1+} can be used for the linearization of the curve and d_{2+} is used in the domain $v_{sw} \dots v_{max}$. The maximum approximation error occurs at the velocity v_{sw} for both linearizations.

If we consider the positive part of the friction model (4), the obtained equations for the d_{1+} and d_{2+}, using Taylor expansion, are:

$$\begin{aligned} d_{1+} &: F_{L1f+}(v) = F_S + \left.\frac{\partial F_f(v)}{\partial v}\right|_{v=0} v \\ &= F_S + (F_V - (F_S - F_C)/v_S)v \qquad (11) \\ d_{2+} &: F_{L2f+}(v) = F_f(v_{max}) + \frac{\partial F_f(v_{max})}{\partial v}(v - v_{max}) \\ &= F_f(v_{max}) + \qquad (12) \\ &\quad (F_V - (F_S - F_C)/v_S)e^{-v_{max}/v_S}(v - v_{max}) \end{aligned}$$

Thus the linearization of the exponential friction model with bounded error can be described by two lines in the $0 \dots v_{max}$ velocity domain:

$$\begin{aligned} d_{1+} &: F_{L1f+}(v) = a_{1+} + b_{1+}v, \\ &\quad for\ 0 \leq v \leq v_{sw} \qquad (13) \\ d_{2+} &: F_{L2f+}(v) = a_{2+} + b_{2+}v, \\ &\quad for\ v_{sw} \leq v \leq v_{max} \qquad (14) \end{aligned}$$

Now let us consider two exponential membership functions parameterized in the following way:

$$\phi_{1+}(v) = \frac{e^{-\beta(v - v_{sw+})}}{1 + e^{-\beta(v - v_{sw+})}} \quad \phi_{2+}(v) = \frac{1}{1 + e^{-\beta(v - v_{sw+})}} \tag{15}$$

where β is a large positive constant and v_{sw+} is the switching velocity where d_{1+} and d_{2+} meets each other. The value of v_{sw+} can easily be determined from linearization (13):

$$\begin{aligned} v_{sw+} &= \frac{a_1 - a_2}{b_2 - b_1} = \qquad (16) \\ &\frac{F_S + F_f(v_{max}) + (F_V - (F_S - F_C)/v_s)e^{-v_{max}/v_s}}{(F_V - (F_S - F_C)/v_s)(-1 + e^{-v_{max}/v_s})} \end{aligned}$$

If we apply the F_{L1f+} from (13) on the membership function ϕ_1 from (15) and F_{L2f+} on ϕ_2 we can obtain a new model that has the same behavior as the Tustin friction model. Moreover it is linearly parameterized if we consider the parameters of the lines. So for the positive velocity domain we have:

$$\begin{aligned} F_{f+}(v) = a_{1+}\phi_{1+}(v)\mu(v) + b_{1+}v\phi_{1+}(v)\mu(v) + \\ a_{2+}\phi_{2+}(v)\mu(v) + b_{2+}v\phi_{2+}(v)\mu(v) \quad (17) \end{aligned}$$

With same train of thoughts a similar model can be determined for the negative velocity domain. Combining the negative and positive velocity domains the obtained friction model reads as:

$$F_f(v) = \underline{\theta}_f^T \underline{\xi}_f(v) \tag{18}$$

$$where: \ \underline{\theta}_f = (a_{1+}\ b_{1+}\ a_{2+}\ b_{2+}\ a_{1-}\ b_{1-}\ a_{2-}\ b_{2-})^T$$

$$\underline{\xi}_f(v) = (\phi_{1+}\mu(v)\ v\phi_{1+}\mu(v)\ \phi_{2+}\mu(v)\ v\phi_{2+}\mu(v)..$$

$$\phi_{1-}\mu(-v)\ v\phi_{1-}\mu(-v)\ \phi_{2-}\mu(-v)\ v\phi_{2-}\mu(-v))^T$$

III. ADAPTIVE COMPENSATION OF THE FRICTION FORCE IN A DC SERVO DRIVE

To illustrate the applicability of the introduced friction model a mechanical positioning system were considered. Let us consider that the mechanical load is driven by a DC servo motor through a gear-head. The friction force acts on the load, inside the gear-head and inside the DC motor.

To determine the equation of motion of the DC servo motor the rotor current dynamics was neglected. This can be made when the electrical time constant (L/R) is sufficiently small (near 1 ms) relative to the used sampling interval in the control algorithm. In this case the rotor current (i) in the function of control voltage (u) and the rotor angular speed (ω) can be written as:

$$i = (u - c_1\omega)/R \tag{19}$$

The equations of motion of the motor using (19) is given by:

$$\begin{aligned} J_R\dot{\omega} &= c_2 i - \tau_{fR} - \tau_{ext} = -A\omega + Ku - \tau_{fR} - \tau_{ext} \\ \dot{\alpha} &= \omega \end{aligned} \tag{20}$$

α denotes the rotor angular speed, τ_{fR} - friction force which acts inside the motor, τ_{ext} - external torque. The angular position and velocity of the motor are known from measurements.

The motor parameters are $J_R > 0$ - the rotor inertia, R - rotor terminal resistance, L - rotor terminal inductance, c_1 - the inverse of speed constant, c_2 - torque constant, $A = c_2c_1/R > 0$, $K = c_2/R > 0$. These constants are catalog data for a specific motor and considered to be known.

The external torque that acts on the rotor can be obtained from:

$$\tau_{ext} = \tau_{fG} + \tau_{fL}/N + J_L\dot{\omega}_L/N + d \tag{21}$$

where τ_{fG} - friction force inside the gear-head, τ_{fL} - friction force which acts on the load, J_L is the unknown inertia of the load, N is the gear ratio. d represents a bounded additive disturbance which incorporates unmodelled dynamics, measurement errors and external disturbances ($|d| \leq D_M$ with D_M known).

From (20) and (21) we obtain the equation of motion of the positioning system:

$$J\ddot{\alpha} + A\dot{\alpha} = Ku - \tau_f + d \tag{22}$$

where $J = J_R + J_G + J_L/N$ in which J_G denotes the unknown inertia introduced by the components of the gear and $\tau_f = \tau_{fR} + \tau_{fG} + \tau_{fL}/N$ is the sum of all friction forces that act on the mechanics. The friction force and the load inertia in the dynamics are considered unknown.

Let us define the tracking error $e(t) = \alpha(t) - \alpha_d(t)$ and the tracking error metric $S(t) = (\frac{d}{dt} + \lambda)e(t)$ with $\lambda > 0$. α_d is the prescribed trajectory, a smooth, twice differentiable function in time [3].

The control problem can be formulated as follows: design a control law u such as that the tracking error metric $S(t)$ satisfies $|S(t)| < \Phi$ for $t \to \infty$ where Φ is a given precision.

From (22) and the expression of the tracking error $S(t)$ we obtain the tracking error dynamics as follows:

$$J\dot{S} = J\ddot{\alpha}_d - \lambda\dot{e} - A\dot{\alpha} + Ku - \tau_f + d \tag{23}$$

where friction force τ_f is modelled using the friction model (18) introduced in Section I. $\tau_f = \underline{\theta}_f^T \underline{\xi}_f(\omega)$

The parameters $\underline{\theta}_f$, J are unknown, consequently the control law can be developed using estimated parameters, that are generated on-line by an adaptation rule. Let us denote the estimation errors and the estimated parameters as follows: $\widetilde{\underline{\theta}}_f = \underline{\theta}_f - \widehat{\underline{\theta}}_f$, $\widetilde{J} = J - \widehat{J}$

To solve the proposed control problem, let us define the following control law:

$$\begin{aligned} u = \frac{1}{K}(A\dot{\alpha} - \widehat{J}(\ddot{\alpha}_d - \lambda\dot{e}(t)) + \widehat{\underline{\theta}}_f \underline{\xi}_f(\omega) - k_S S_\Delta(t) \\ - D_M sat(S/\Phi)) \end{aligned} \tag{24}$$

with $k_S > 0$ and $S_\Delta(t) = S(t) - sat(S(t)/\Phi)$ where $sat(\cdot)$ denotes the saturation function. The following propriety can be easily verified:

$$\dot{S}_\Delta = \dot{S} \ for \ |S_\Delta| \geq \Phi \ and \ \dot{S}_\Delta = 0 \ otherwise. \tag{25}$$

The values of the friction and inertial parameters can be obtained using adaptive techniques. The most common adaptation rule in the adaptive control systems is the gradient method [3]. To increase the robustness of the control system a modified gradient algorithm, the switching-σ adaptation law [3] is applied in this paper:

$$\begin{aligned} \dot{\widehat{\theta}}_{fi} &= -\gamma_{\theta_{fi}}\xi_{fi}(\omega)S_\Delta(t) - \sigma(\widehat{\theta}_i)\gamma_{\theta_{fi}}\widehat{\theta}_i \\ \dot{\widehat{J}} &= -\gamma_J(\ddot{\alpha}_d - \lambda\dot{e}(t))S_\Delta(t) - \sigma(\widehat{J})\gamma_J\widehat{J} \end{aligned} \tag{26}$$

with γ_{θ_i}, γ_J strictly positive adaptation gains.

The function σ is defined as:

$$\sigma(\widehat{\theta}_i) = \begin{cases} 0, & \text{if } \Theta_i \leq |\widehat{\theta}_i| \\ \sigma_0(|\widehat{\theta}_i|/\Theta_i - 1), & \text{if } |\widehat{\theta}_i| < \Theta_i \leq 2|\widehat{\theta}_i| \\ \sigma_0, & \text{otherwise} \end{cases} \tag{27}$$

where $\sigma_0 > 0$ and $\Theta_i > |\theta_i|$.

The closed loop system with the control law (24) for $S \geq \Phi$ can be written as:

$$\begin{aligned} J\dot{S}(t) = \widetilde{J}(\ddot{\alpha}_d - \lambda\dot{e}(t)) - \widetilde{\underline{\theta}}_f^T \underline{\xi}_f(\omega) - k_S S_\Delta(t) \\ -(D_M sat(S/\Phi) - d) \end{aligned} \tag{28}$$

To examine the behavior of the closed loop system let us consider the following Lyapunov like cost function:

$$V(t) = JS_\Delta(t)^2 + \frac{1}{\gamma_J}\widetilde{J}^2 + \sum_{i=1}^{8}\frac{1}{\gamma_{\theta_{fi}}}\widetilde{\theta}_{fi}^2 \quad (29)$$

The time derivative of the Lyapunov function is given by:

$$\dot{V}(t) = J\dot{S}_\Delta(t)S_\Delta(t) + \frac{1}{\gamma_J}\dot{\widetilde{J}}\widetilde{J} + \sum_{i=1}^{8}\frac{1}{\gamma_{\theta_{fi}}}\dot{\widetilde{\theta}}_{fi}\widetilde{\theta}_{fi} \quad (30)$$

It can be shown [3] that $\sigma(\widehat{\theta}_i)\widehat{\theta}_i\widetilde{\theta}_i \leq 0$. Moreover, exploiting the property (25) we have $\dot{V}(t) \leq 0$ if $S(t) < \Phi$. If $S(t) \geq \Phi$ the equation of the closed loop system can be introduced in (30). If we also introduce the adaptation laws (26) we obtain:

$$\begin{aligned}\dot{V}(t) = & - k_S S_\Delta(t)^2 - S_\Delta(t)(D_M sat(S/\Phi) - d) \\ & + \sigma(\widehat{J})\widehat{J}\widetilde{J} + \sum_{i=1}^{8}\sigma(\widehat{\theta}_{fi})\widehat{\theta}_{fi}\widetilde{\theta}_{fi} \quad (31)\end{aligned}$$

Because it was assumed that $S(t) \geq \Phi$ we have $sat(S/\Phi) = sign(S) = sign(S_\Delta)$. Using the following simple relation that if $|d| < D_M \Rightarrow dS_\Delta \leq D_M |S_\Delta|$.
At the other hand if $\sigma_0 > 0$ from the definition (27) results $\sigma(\widehat{\theta}_i)\widehat{\theta}_i\widetilde{\theta}_i \leq 0$. From these observations yields:

$$\dot{V}(t) \leq -k_S S_\Delta(t)^2 \quad (32)$$

Notice that (32) is also valid for $|S(t)| < \Phi$. Since $V(t)$ is a positive and non-increasing function, therefore $V(\infty)$ is finite and well defined.
Thus, if $S_\Delta(0)$, $\widetilde{J}(0)$ and $\widetilde{\underline{\theta}}_f(0)$ is bounded $\Rightarrow S_\Delta(t)$, $\widetilde{J}(t)$ and $\widetilde{\underline{\theta}}_f(t) \in L_\infty \; \forall \; t > 0$.
If $S_\Delta(t)$, $e(0)$ and $\dot{e}(0)$ is bounded $\Rightarrow e(t)$ and $\dot{e}(t) \in L_\infty$.
If $e(t)$, $\dot{e}(t)$, $\alpha_d(t)$ and $\dot{\alpha}_d(t) \in L_\infty \Rightarrow \alpha(t)$, $\dot{\alpha}(t) \in L_\infty$.
From (24) results that if $\widehat{J}$, $\widehat{\underline{\theta}}_f$, $\ddot{\alpha}_d$, $\dot{e}(t)$ and $S_\Delta \in L_\infty$ $\Rightarrow u(t) \in L_\infty$.
From (28) results that if $S_\Delta(t)$, $\widetilde{J}(t)$, $\widetilde{\underline{\theta}}_f(t)$, $\ddot{\alpha}_d(t)$, $\dot{e}(t)$ $u(t) \in L_\infty \Rightarrow \dot{S}_\Delta(t) \in L_\infty$.
From:

$$\int_0^\infty S_\Delta(t)^2 dt \leq \frac{-1}{k_S}\int_0^\infty \dot{V}(t) = \frac{V(0) - V(\infty)}{k_S} < \infty \quad (33)$$

results that $S_\Delta(t) \in L_2$.
Because $S_\Delta(t)$ and $\dot{S}_\Delta(t) \in L_\infty$ and the relation (33) holds, by Barbalat's lemma $S_\Delta(t) \to 0$ when $t \to \infty$, consequently the inequality $|S(t)| \leq \Phi$ is obtained asymptotically. Thus the control law (24) with the adaptation law (26) solves the formulated control problem.

IV. SIMULATIONS AND RESULTS

In order to test the applicability of our theoretical results we performed simulations on a DC motor to which a load is attached through a gearbox. The inertial load is also in contact with an external surface, which gives an external frictional force. For the motor the following parameters were used: $J_R = 41.9 \times 10^{-7}\ kg/m^2$, $A = 1.508 rad/secV$, $K = 1 rad/secNm$. The gear ratio was taken as $N = 66$ and the external inertia $J = 0,001 kgm^2$. The friction force was modelled using the relation (7) with the following parameters: $F_S = 0.0015\ Nm$, $F_C = 0.001\ Nm$, $F_V = 0.01\ Nmsec/rad$, $v_S = 100\ rad/sec$.
The prescribed trajectory is a sinusoidal one with small amplitude which assures that the mechanical system moves both in positive and negative velocity regime near zero velocities.

$$\dot{\alpha}_d(t) = \omega_d(t) = sin(10t)\ [rad/sec]$$

$$\alpha_d(t) = \int_0^t \omega(\tau)d\tau\ [rad];\ \ddot{\alpha}_d(t) = \frac{d\omega(t)}{dt}\ [rad/sec^2]$$

The control objective is to track this prescribed position, such that the tracking error metric $S(t) \leq 10^{-3}$. The parameters of the controller were chosen as follows: $\lambda = 10$, $k_S = 20$, $D_M = 1E-4$, $\Phi = 1E-3$.
The friction was modelled in the control law using the relation (18). All parameters of the friction model was departed with at least 50% from its real values. The simulation results in Fig. 7 and 10 shows that the control law guarantees very precise tracking for position output. The convergence of the friction parameters in the positive velocity regimes during the adaptation is also presented (Fig. 5, 6, 8, 9). Due to membership functions (15) it can be observed that the parameters determining the behavior of friction force are tuned only when the plant is in the corresponding velocity regime.

V. CONCLUSIONS

The paper deals with the problem of adaptive compensation of friction force in DC servo motor controlled mechanical positioning systems. A novel friction model for low velocity regime is presented which has the useful property that it can be written in a linearly parameterized form, hence the well known parameter estimation algorithms can be applied to determine its parameters. The proposed adaptive tracking control algorithm which incorporates the introduced friction model guarantees that the tracking error remains bounded with known bound. Simulations shows good tracking performances and parameter convergence of the control law.

REFERENCES

[1] Brian Armstrong-Hèlouvry, *Control of Machines with Friction*, Kluver Academic Press, Boston, 1991.

[2] C. Canudas de Wit, "Control of systems with dynamic friction", in *CCA Workshop on Systems with Friction*, 1999.

[3] Petros A. Ioannou and Jing Sun, *Robust Adaptive Control*, Prentice Hall, Upper Slade River, NJ, 1996.

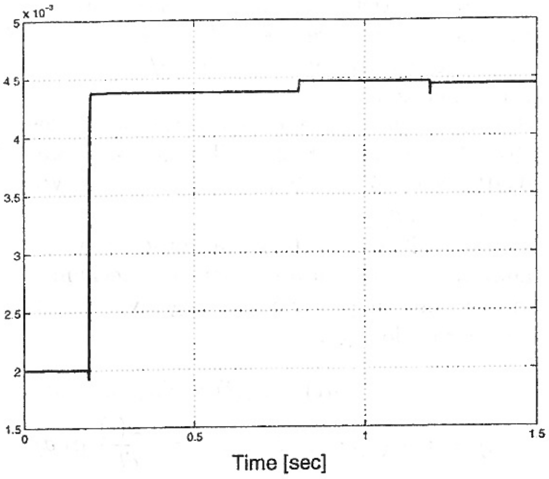

Fig. 5: Convergence of parameter a_{1+}

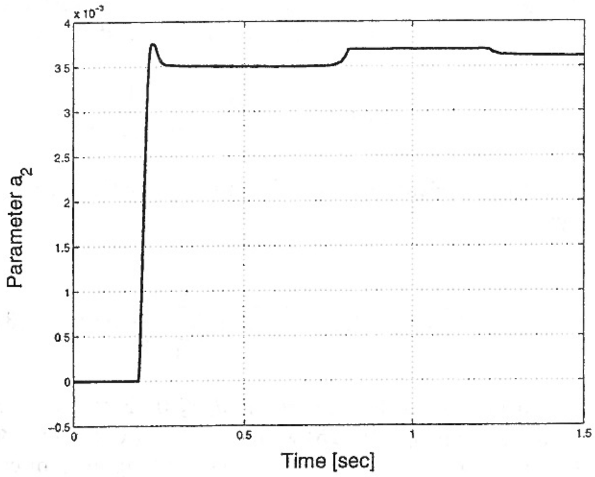

Fig. 8: Convergence of parameter a_{2+}

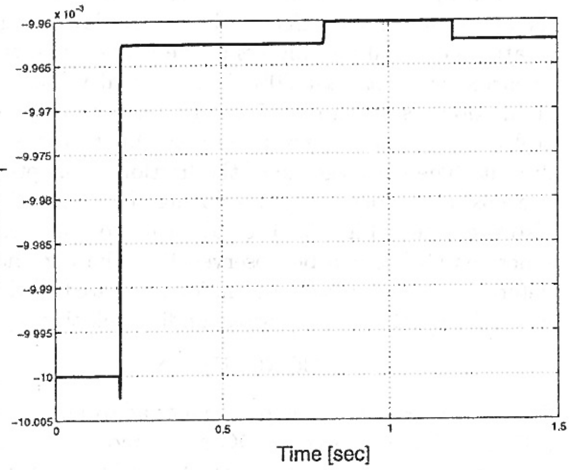

Fig. 6: Convergence of parameter b_{1+}

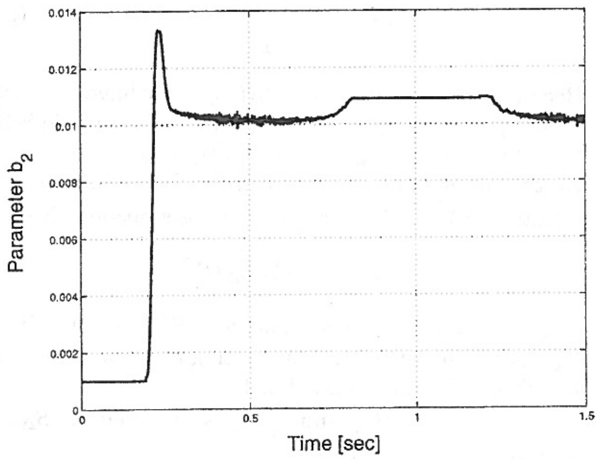

Fig. 9: Convergence of parameter b_{2+}

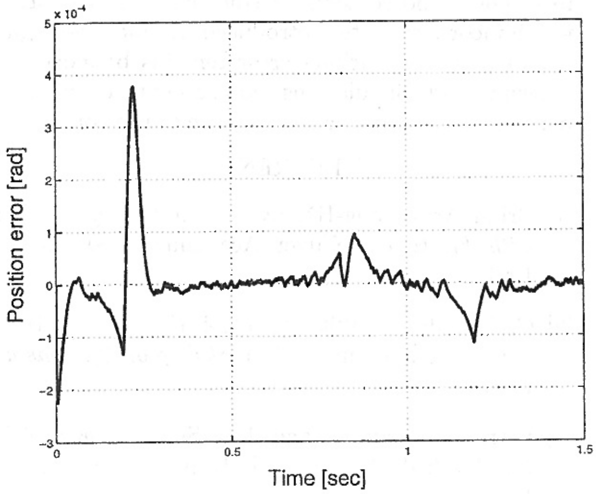

Fig. 7: Position tracking error

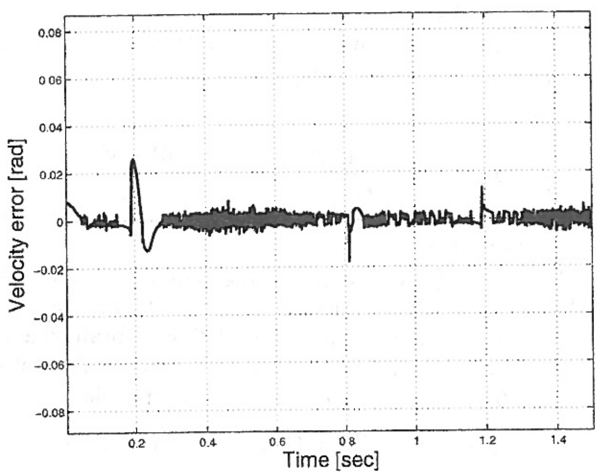

Fig. 10: Velocity tracking error