Application of Extended Numerical Approximation of Fractional Order Derivatives in Adaptive Control

József K. Tar, János F. Bitó
Institute of Mathematics and Computational Sciences
Budapest Polytechnic

Népszínház u. 8, H-1081 Budapest

Hungary
{tar.jozsef, bito.janos}@nik.bmf.hu

José A. Tenreiro Machado
Department of Electrotechnical Engineering
Institute Engineering of Porto

Rua Dr. Antonio Bernardino de Almeida 4200-072 Porto
Portugal
jtm@dee.isep.ipp.pt

Domonkos Tikk
Dept. of Telecommunications & Telematics
Budapest University of Technology and Economics
Magyar Tudósok Körútja 2., H-1117 Budapest
Hungary
tikk@ttt.bme.hu

Abstract – **In a novel branch of soft computing developed in the past few years the desired and the expected response of the system is mapped to each other. In the case of mechanical systems the compared values are the second time-derivatives of the joint coordinates for the estimation of which certain finite element approximations are used in a digital control. This may result in a kind of noise and estimation sensitivity. In the present paper these integer order derivatives are replaced by discrete numerical estimations of fractional order derivatives near the order of two to make the control more stable and accurate. For this purpose Caputo's form is considered the numerical approximation of which can be extended over the limits of the original definition. In this view differentiation seems to be an operation with some time-invariant Green function. Simulation results obtained for the adaptive control of an inaccurately modeled electromechanical system containing an unmodeled and undriven internal degree of freedom illustrate that the quality of the control can be improved if the order of derivation in the signals used fro comparison are increased from 2 to 2.25.**

I. INTRODUCTION

Adaptive control of various physical systems may have the „delicate" nature that certain internal degrees of freedom of these systems are neither observable nor directly driven, so the cannot be controlled. However, the physical states of these subsystems influence the motion of the observed and controlled (that is actuated) ones via nonlinear coupling.

Realizing that "generality" and "uniformity" of the "traditional SC structures" exclude the application of plausible simplifications made the idea rise that by addressing narrower problem classes a novel branch of soft computing could be developed by the use of far simpler and far more lucid uniform structures and procedures than the classical ones. On the basis of the simultaneous use of the Modified Renormalization Transformation and simple ancillary methods [1] it is flexible enough to incorporate various special groups serving as the resources of the uniform structures, and to apply special algebraic blocks for learning as Lorentz Transformations [2], and Minimum Operation Symplectic Transformations [3] etc. The main idea of this approach is the construction of a simple mapping that maps the observed behavior of the controlled system to the desired one calculated on the basis of a rough, approximate, incomplete model. Originally it was elaborated via considering the internal symmetries of Classical Mechanical Systems [5] on the basis of [4]. It is interesting that the Canonical Formalism of Classical Mechanics offers various advantages revealed and utilized recently, too [6-7].

In the case of Classical Mechanical Systems the observable and directly controllable agents are the second time-derivatives of the joint coordinates. The double integer order derivation can introduce noise-sensitivity into such systems especially if it is implemented via the application of discrete-time finite element resolution.

As the generalization of the operation of derivation the concept of fractional order integrals and derivatives found more and more physical applications in describing the „longer term memory" of various physical systems as e.g. in the case of visco-elastic phenomena [8-9], seismic analysis [10], or in the case of control technology [11], etc. Fractional order derivatives obtained various definitions from 19th century [12, 13, or 14]. In general these operators have an effect similar to integration with some Green function. In the sequel first this analogy is highlighted, following that the adaptive control is described, simulations results are given and analyzed and the conclusions are drawn.

II. FRACTIONAL ORDER DERIVATIVES AND GREEN FUNCTIONS

For our purposes the definition of the fractional order derivatives given by Caputo seems to be the most expedient as

$$\frac{d^\beta}{dt^\beta}u(t) := \frac{1}{\Gamma(1-\beta)}\int_0^t \left[\frac{du(\tau)}{d\tau}\right](t-\tau)^{-\beta}d\tau\,,\ \beta\in(0,1) \qquad (1)$$

For $t>0$ (1) physically has the following simple meaning: the full 1st order derivative in the integrand removes the constant component from the signal, and this derivative is "causally reintegrated" by the use of a Green function that has a slowly forgetting nature (the contribution of the far past becomes more and more negligible in it), while its singularity in $\tau=t$ enhances the relative weight of the contribution of the $\tau\lessapprox t$ instants. Furthermore, the relatively slowly decreasing "tail" of this function also acts as a frequency filter that rejects the high-frequency components of the traditional 1st derivative.

Due to the singularity of the Green function in (1) a common finite-element numerical integration cannot accurately be done. Instead of that, we can suppose that at least $u'(\tau)$ is a relatively slowly varying function of time, therefore it can be considered as constant during the integration over a small time-interval, while the variation of the Green function can be taken into account accurately. Furthermore, to introduce symmetry against the translation of the signal in time we can omit the very long tail of the Green-function and we can go back in time only to some time $t\text{-}T$ instead of 0. The proposed approximation of (1) in this paper was taken as

$$\frac{d^{\beta}}{dt^{\beta}}u(t)\cong\frac{u'(t)\delta^{-\beta+1}}{\Gamma(2-\beta)}+ \\ +\sum_{0<s\ \text{while}\ s\delta<T}\frac{\delta^{-\beta+1}\left[(s+1)^{-\beta+1}-s^{-\beta+1}\right]}{\Gamma(2-\beta)}u'(t-s\delta) \quad (2)$$

In the present approach it is not our purpose to obtain exact integer order derivatives from this approximation when $\beta\to1$ or $\beta\to0$. Our aim is simply the use of some weighted average of the past signal. Furthermore, the numerical approximation (2) can be extended to $\beta\geq1$ for which (1) is not defined. Fig. 1 describes the coefficients in (2) for $\beta=1.25$ and $\delta=10^{-3}$ s for 50 points.

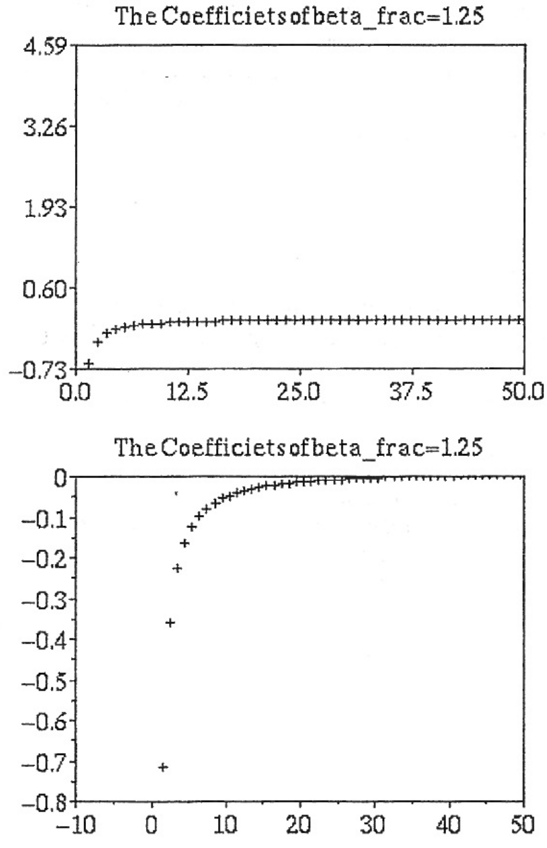


Fig. 1. The coefficients of the "extended" numerical approximation of the Caputo form for $\beta=1.25$ and $\delta=10^{-3}$ s (full scale and zoomed excerpt for the "tail").

It is easy to see that the calculations made for (2) for $\beta=1$ just gives the first derivative, while for $0<\beta<1$ the character of the function of coefficients vs. time considerably differs from that described in Fig. 1.

In the present paper Caputo's definition for $n+\beta$ [$\beta\in(0,1)$, $n=0,1,2,\ldots$] is applied with the numerical approximation given in (4), therefore $\beta=1.25$, and $n=1$ results in the "2.25th" derivative in the simulations.

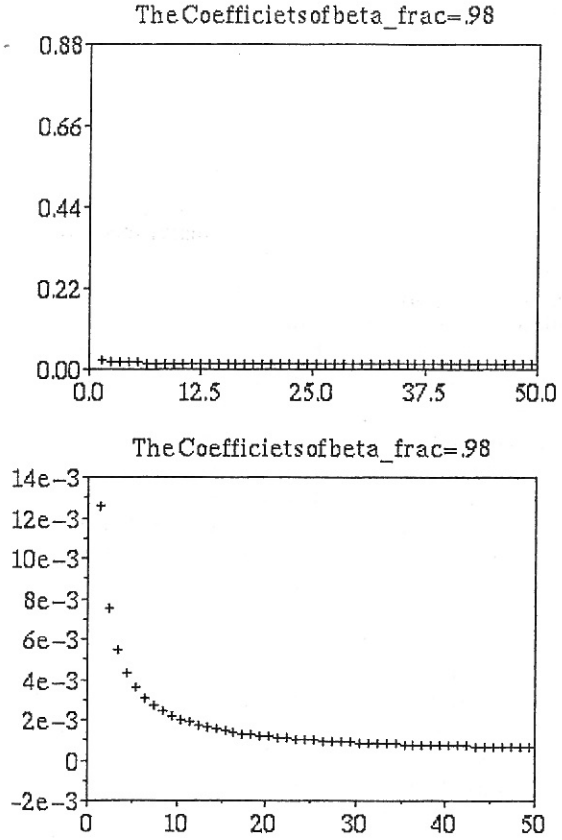


Fig. 2. The coefficients of the numerical approximation of the Caputo form for $\beta=0.98$ and $\delta=10^{-3}$ s (full scale and zoomed excerpt for the "tail").

$$\frac{d^{n+\beta}}{dt^{n+\beta}}u(t):=\frac{1}{\Gamma(1-\beta)}\int_0^t\left[\frac{d^{n+1}u(\tau)}{d\tau^{n+1}}\right](t-\tau)^{-\beta}d\tau \quad (3)$$

$$\frac{d^{n+\beta}}{dt^{n+\beta}}u(t)\cong\frac{u^{(n+1)}(t)\delta^{-\beta+1}}{\Gamma(2-\beta)}+ \\ +\sum_{0<s\ \text{while}\ s\delta<T}\frac{\delta^{-\beta+1}\left[(s+1)^{-\beta+1}-s^{-\beta+1}\right]}{\Gamma(2-\beta)}u^{(n+1)}(t-s\delta) \quad (4)$$

III. PRINCIPLES OF THE ADAPTIVE CONTROL

For the adaptive control there is given an imperfect system model as a starting point. On the basis of that some excitation is calculated to obtain a desired system response $\mathbf{i}^d$ as $\mathbf{e}=\varphi(\mathbf{i}^d)$. This model is step by step refined in the following manner. If we apply the above approximate excitation, according to the actual system's inverse dynamics described by the unknown function a realized response $\mathbf{i}^r=\psi(\varphi(\mathbf{i}^d))=f(\mathbf{i}^d)$ is obtained instead of the desired one, $\mathbf{i}^d$. Normally one can obtain information via observation only on the function $f()$ considerably varying in time, and no any possibility exists to directly "manipulate" the nature of this function: only $\mathbf{i}^d$ as the input of $f()$ can be "deformed" to $\mathbf{i}^{d*}$ to achieve and maintain the $\mathbf{i}^d=f(\mathbf{i}^{d*})$ state. [Only the *model function* $\varphi()$ can directly be manipulated.] On the basis of the modification of the method of renormalization widely applied in Physics the following "scaling iteration" was suggested for finding the proper deformation:

$$\mathbf{i}_0; \mathbf{S}_1\mathbf{f}(\mathbf{i}_0) = \mathbf{i}_0; \mathbf{i}_1 = \mathbf{S}_1\mathbf{i}_0; \ldots; \mathbf{S}_n\mathbf{f}(\mathbf{i}_{n-1}) = \mathbf{i}_0; \quad \mathbf{i}_{n+1} = \mathbf{S}_{n+1}\mathbf{i}_n; \mathbf{S}_n \xrightarrow[n\to\infty]{} \mathbf{I} \tag{5}$$

in which the $\mathbf{S}_n$ matrices denote some linear transformations to be specified later. As it can be seen these matrices maps the observed response to the desired one, and the construction of each matrix corresponds to a step in the adaptive control. It is evident that if this series converges to the identity operator just the proper deformation is approached, therefore the controller „learns" the behavior of the observed system by step-by-step amendment and maintenance of the initial model. Since (5) does not unambiguously determine the possible applicable quadratic matrices, we have additional freedom in choosing appropriate ones. The most important points of view are fast and efficient computation, and the ability for remaining as close to the identity transformation as possible. For making the problem mathematically unambiguous (5) can be transformed into a matrix equation by putting the values of **f** and **i** into well-defined blocks of bigger matrices. Via computing the inverse of the matrix containing **f** in (5) the problem can be made mathematically well defined. Since the calculation of the inverse of one of the matrices is needed in each control cycle it is expedient to choose special matrices of fast and easy invertibility. Within the block matrices the response arrays may be extended by adding to them a "dummy", that is physically not interpreted dimension of constant value, in order to evade the occurrence of the mathematically dubious 0→0, 0→finite, finite→0 transformations. In the present paper the Minimum Operation Symplectic Matrices announced in [3] were applied for this purpose.

In general, the Lie group of the Symplectic Matrices is defined by the equations

$$\mathbf{S}^T\mathfrak{I}\mathbf{S} = \mathfrak{I} \equiv \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix}, \ \det\mathbf{S} = 1 \tag{6}$$

The inverse of such matrices can be calculated in a computationally very cost-efficient manner as $\mathbf{S}^{-1} = \mathfrak{I}^T\mathbf{S}^T\mathfrak{I}$. In our particular case

$$\mathbf{S} \equiv \left[\begin{array}{cccc|cccc} \mathbf{f}^{(1)} & \mathbf{u}^{(2)} & \mathbf{e}^{(3)} & \ldots & -\tilde{\mathbf{f}}^{(2)} & -\tilde{\mathbf{u}}^{(1)} & 0 & \ldots \\ \mathbf{f}^{(2)} & \mathbf{u}^{(1)} & 0 & \ldots & \tilde{\mathbf{f}}^{(1)} & \tilde{\mathbf{u}}^{(2)} & \mathbf{e}^{(3)} & \ldots \end{array}\right] \tag{7}$$

in which $\mathbf{f}^{(1)}, \mathbf{f}^{(2)} \in \Re^{DOF}$ two linearly independent non-zero vectors constructed of a "dummy parameter" d used for avoiding the occurrence of 0→0 type mapping as $\mathbf{f} = \begin{bmatrix}\mathbf{f}^{(1)T} & \mathbf{f}^{(2)T}\end{bmatrix}^T = [\ddot{q}_1/w \quad d \quad -d \quad \ddot{q}_1/w]^T$, whenever the joint coordinate accelerations are taken into account as the response of the system. In this paper the main idea is that instead of the 2nd order derivative we use the fractional order derivatives for comparison in the above given sense:

$$\mathbf{f} = \begin{bmatrix}\mathbf{f}^{(1)T} & \mathbf{f}^{(2)T}\end{bmatrix}^T = \left[q_1^{(1+\beta)}/w \quad d \quad -d \quad q_1^{(1+\beta)}/w\right]^T. \tag{8}$$

In (7) the orthonormal set $\{\mathbf{e}^{(j)} \in \Re^{DOF} \mid j = 2,3,\ldots,DOF\}$ can arbitrarily be chosen in the orthogonal subset of $\{\mathbf{f}^{(1)}, \mathbf{f}^{(2)}\}$. (Parameter w means an adaptive forgetting weight factor that is given later.) The other components of the matrix are defined as follows:

$$\mathbf{u}^{(1)} = \frac{1}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}}\left[\mathbf{f}^{(1)} - \frac{\mathbf{f}^{(2)T}\mathbf{f}^{(1)}}{\mathbf{f}^{(2)T}\mathbf{f}^{(2)}}\mathbf{f}^{(2)}\right]$$
$$\mathbf{u}^{(2)} = \frac{1}{\mathbf{f}^{(2)T}\mathbf{f}^{(2)}}\left[\mathbf{f}^{(2)} - \frac{\mathbf{f}^{(1)T}\mathbf{f}^{(2)}}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}}\mathbf{f}^{(1)}\right] \tag{9}$$

$$\tilde{\mathbf{f}}^{(1)} = \frac{\mathbf{f}^{(1)}}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)} + \mathbf{f}^{(2)T}\mathbf{f}^{(2)}}, \ \tilde{\mathbf{f}}^{(2)} = \frac{\mathbf{f}^{(2)}}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)} + \mathbf{f}^{(2)T}\mathbf{f}^{(2)}} \tag{10}$$

$$\tilde{\mathbf{u}}^{(1)} = \frac{\mathbf{u}^{(1)}}{\mathbf{u}^{(1)T}\mathbf{u}^{(1)} + \mathbf{u}^{(2)T}\mathbf{u}^{(2)}}, \ \tilde{\mathbf{u}}^{(2)} = \frac{\mathbf{u}^{(2)}}{\mathbf{u}^{(1)T}\mathbf{u}^{(1)} + \mathbf{u}^{(2)T}\mathbf{u}^{(2)}} \tag{11}$$

Both the desired and the observed accelerations generate their own symplectic matrices. One of them must be inverted to produce a solution used in (5). The unit vectors can be created e.g. by using El Hini's algorithm (details are given e.g. in [15]). Since amongst the conditions for which the convergence of the method was proved in [1] near-identity transformations were supposed in the perturbation theory, a parameter ξ measuring the „extent of the necessary transformation", a „regulation factor" λ can be introduced in a linear interpolation with small positive ε_1, ε_2 values as

$$\xi := \frac{\left|\mathbf{f} - \mathbf{i}^d\right|}{\max(|\mathbf{f}|, |\mathbf{i}^d|)}, \quad \lambda = 1 + \varepsilon_1 + (\varepsilon_2 - 1 - \varepsilon_1)\frac{\xi}{1+\xi}, \quad \hat{\mathbf{i}}^d = \mathbf{f} + \lambda(\mathbf{i}^d - \mathbf{f}) \tag{12}$$

This interpolation reduces the task of the adaptive control in the more critical session and helps to keep the necessary linear transformation in the vicinity of the identity operator. Other important fact concerning the details of the numerical calculations is the ratio of $|\ddot{q}|_1$ and d in (9). The controller has *a priori* information only on the *nominal* accelerations, but for the appropriate error-relaxation much higher *desired* accelerations may occur. For this purpose a slowly forgetting integrating filter was introduced to create a weighting factor for $0<\gamma<1$ as

$$w(t_i) := \sum_{j=0}^{\infty} \gamma^j \left|\ddot{q}^{\mathrm{Des}}(t_{i-j})\right| \Big/ \sum_{s=0}^{\infty} \gamma^s \tag{13}$$

In the forthcoming simulations the following numerical data were used: d=12.5, γ=0.9, ε_1=0, ε_2=10^{-4} were chosen.

IV. THE DYNAMIC MODEL OF THE DC MOTOR DRIVEN PENDULUM

Let the pendulum have the rotational generalized coordinate q_1 [*rad*] rotating a ballast of mass m [*kg*]. The length of the pendulum is the uncontrolled degree of freedom described by the generalized coordinate q_2 [*m*]. The ballast is "fixed" by spring of stiffness k [*N/m*] exerting zero force when $q_2=l$. The Euler-Lagrange equations of motion in which g [m/s^2], Q_1 [$N\times m$], and Q_2 [*N*] denote the gravitational acceleration, the driving torque rotating the pendulum, and the force moving the ballast in the radial direction (it is eqal to zero in our case because it does not have actuation), respecively, are given as follows:

$$
\begin{aligned}
& m q_2^2 \ddot{q}_1 + 2 m q_2 \dot{q}_2 \dot{q}_1 + m g q_2 \sin q_1 = Q_1 \\
& m \ddot{q}_2 - m q_2 \dot{q}_1^2 - m g \cos q_1 + k(q_2 - l) = Q_2 \equiv 0
\end{aligned}
\qquad (14)
$$

Q_1 can be controlled via the traditional computed torque control using a DC motor as:

$$
\begin{aligned}
& \dot{Q}(t) + \frac{R}{L} Q(t) + \frac{\mu^2 K K_b}{L} \dot{q}_m(t) = \frac{\mu K}{L} U(t) \\
& \dot{Q}(t) + A Q(t) + B \dot{q}_m(t) = C U(t)
\end{aligned}
\qquad (15)
$$

in which the appropriate terms have the following physical interpretation:

- $U(t)$ is the motor voltage (provided by a voltage generator, used for control purposes);
- L denotes the armature inductance (constant, characteristic to the coil in the armature);
- R stands for resistance of the armature coil (constant);
- μ is the gear ratio, in our case μ=1;
- K_b is the electromotive self-induction constant;
- K means the torque constant of the DC motor;
- $q_m \equiv q_1$ measures the rotation of the motor's shaft.

If we distinguish between the "exact" motor parameters $\{A,B,C\}$ and their approximately known values $\{\tilde{A}, \tilde{B}, \tilde{C}\}$ in the case of CTC control --that is when the "desired torque" is prescribed-- this leads to the equation of motion as

$$
\dot{Q}^{Real} = \frac{C}{\tilde{C}} \dot{Q}^{Des} + \left(\frac{C}{\tilde{C}} \tilde{A} - A \right) Q^{Real} + \left(\frac{C}{\tilde{C}} \tilde{B} - B \right) \dot{q}_1 \qquad (16)
$$

We can try to use the linear control law for the "hypothetic" torque with a constant parameter α as

$$
\dot{Q}_1^{Des} = \alpha \left(Q_1^{HypDes} - Q_1^{HypNow} \right) \qquad (17)
$$

In the simulations the actual electrical parameters were as A=1, B=1, C=1 and their "model value" were $\tilde{A} = 3, \tilde{B} = 4, \tilde{C} = 0.8$, while α depended on the prescribed trajectory tracking property. According to a rough initial mechanical model the system the necessary torque was computed from the formula:

$$
Q_1^{Hyp} = 0.1 m l^2 \ddot{q}_1^{Des} + 100 \times sign\left(\ddot{q}_1^{Des} \right) \qquad (18)
$$

V. SIMULATION RESULTS

In Fig. 3 the results of trajectory tracking are given for the rough initial model for the non-adaptive control, the adaptive control using the finite order approximation of the 2nd (i.e. integer) order derivatives, ad the adaptive control using the "numerically extended" order of differentiation 2.25 for comparison (mapping) purposes.

It is clear that application of the adaptive control considerably improves the quality of the control, that is the adaptive control can "learn" the behavior of the system to be controlled. The stabilizing effect of the extension of the order of derivation is obvious from the figures, too.

It is even more obvious in the figures describing the phase space of the controlled joint (Fig. 4).

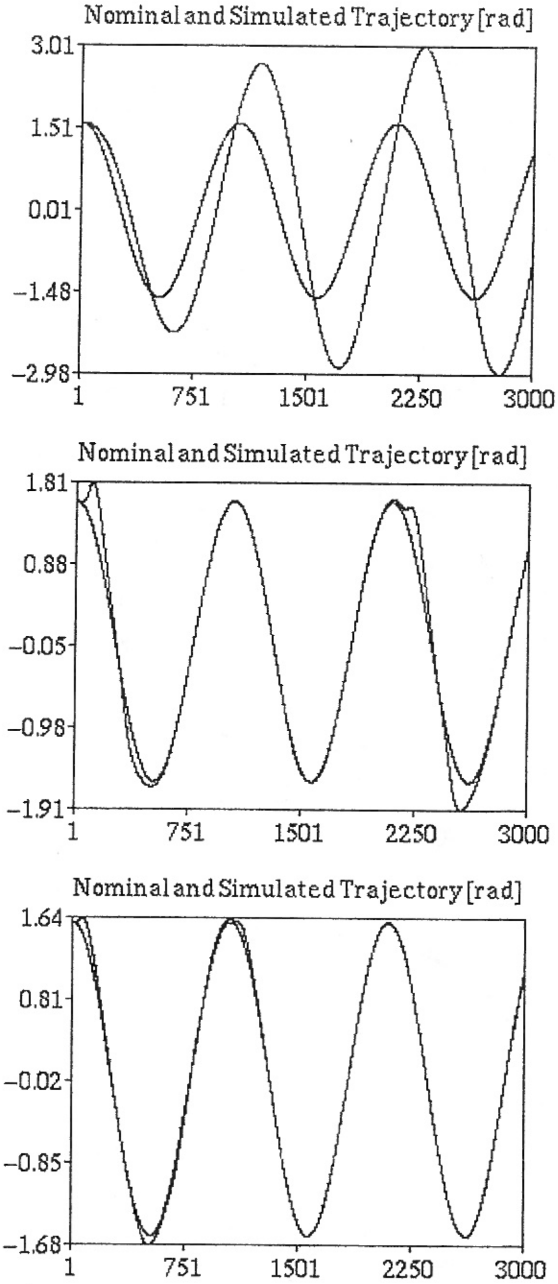


Fig. 3: Trajectory tracking for the non-adaptive control (on the top), the adaptive control with 2nd order derivatives (in the center), and the adaptive control using 2.25 "numerically extended order of derivation" (on the bottom) [*rad*], time in [*ms*].

Fig. 5 describes the phase trajectory of the uncontrolled degree of freedom for the above cases. It is evident that this coupled degree of freedom considerably influenced the dynamic behavior of the controlled and actuated joint so the adaptive control was subject to strong and significant learning requirements.

The efficiency of learning can also be revealed in Fig. 6 describing the torque exerted for the control. The fast variation "superimposed" on the slowly varying component is related to the varying length of the pendulum in the adaptive case.

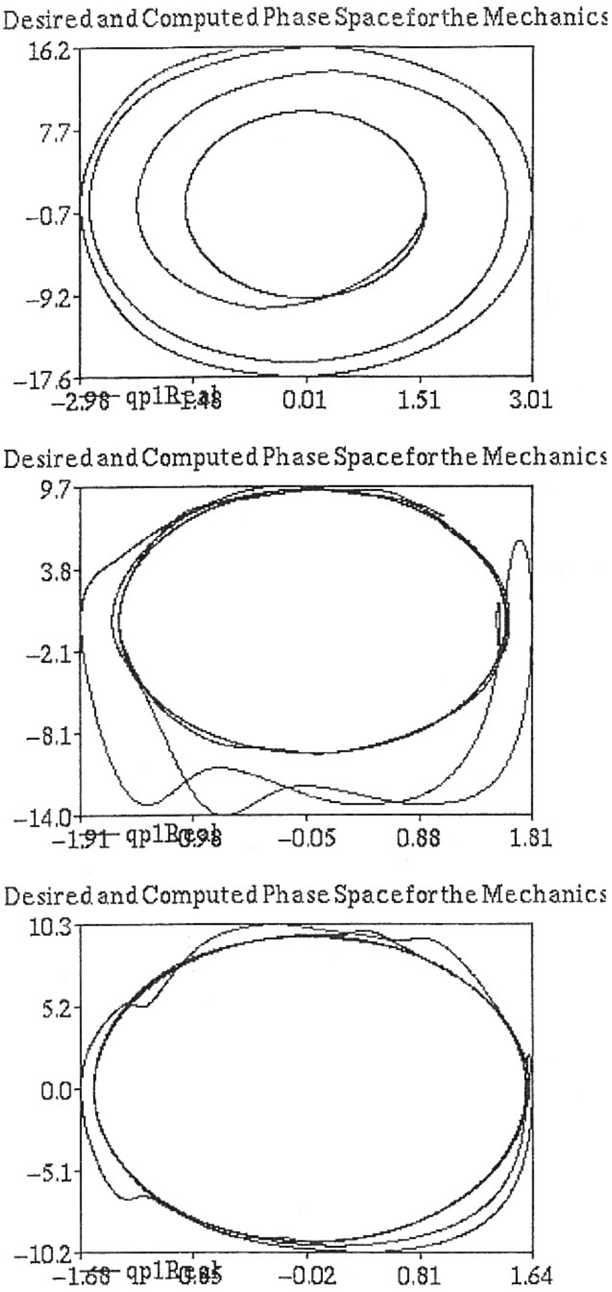


Fig. 4: Phase trajectory of the controlled joint for the non-adaptive control (on the top), the adaptive control with 2nd order derivatives (in the center), and the adaptive control using 2.25 numerically extended order of derivation (on the bottom) [*rad*, *rad/s*].

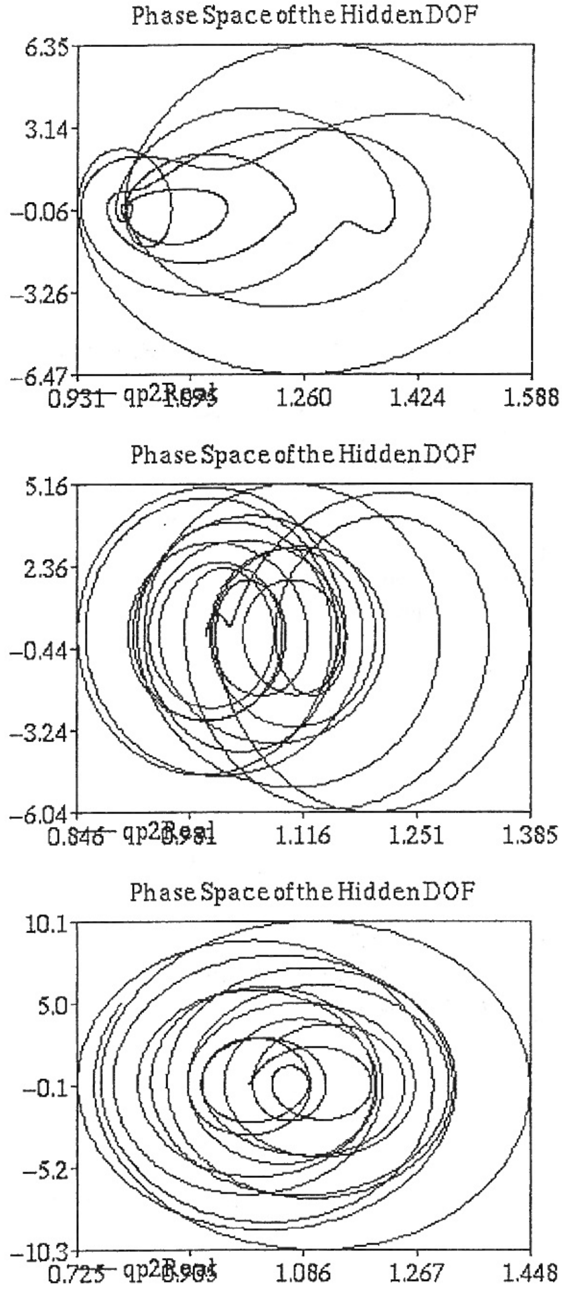


Fig. 5: Phase trajectory of the uncontrolled joint for the non-adaptive control (on the top), the adaptive control with 2nd order derivatives (in the center), and the adaptive control using 2.25 numerically extended order of derivation (on the bottom) [*m*, *m/s*].

VI. CONCLUSIONS

In this paper the extension of a numerical approximation of Caputo s fractional order derivatives was applied for improving the quality of an adaptive control based on a novel branch of soft computing.

The main idea was the replacement of the 2nd order derivatives with extended order ones in the algorithm mapping the observed behavior to the expected one.

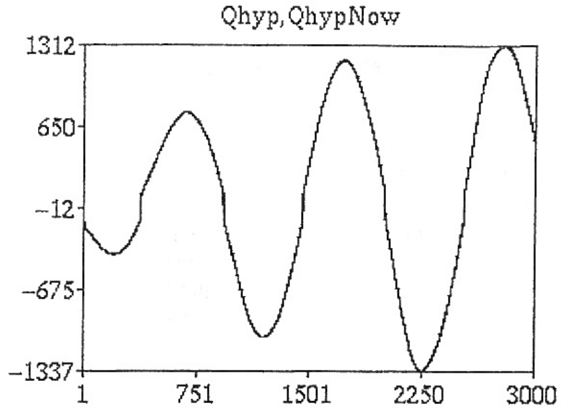
It was found via simulation that the extension of the order of derivation in the given special way from 2 to 2.25 had considerable stabilizing effect when the 2nd order derivative is obtained from the simplest finite element approximation.

The approximation of the extended derivative in this approach took into account the past life of the system to a 50 *ms* time-horizon.

The paradigm used for the simulation investigation contained an uncontrolled degree of freedom (the length of a deformable pendulum) that had very drastic variation in the case of the nominal motion prescribed for the controlled joint, namely for the angle of the pendulum.

Via strong dynamic coupling its actual position and velocity considerably influenced the behavior of the controlled joints.

It seems to be expedient to prove various extensions of



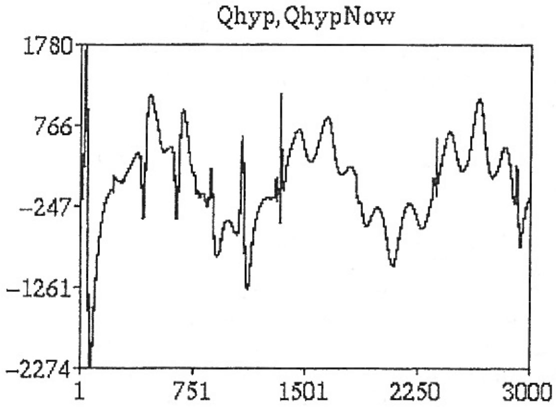


Fig. 6: Exerted torque vs. time for the non-adaptive control (on the top), and the adaptive control using 2.25 "numerically extended order of derivation" (on the bottom) [*Nm*], time in [*ms*].

of the order of derivation in the mapping phase of the adaptive controller for stabilizing the control in the case of various physical systems containing free, unmodeled and uncontrolled internal degree(s) of freedom.

VII. ACKNOWLEDGMENT

The authors gratefully acknowledge the support by the Hungarian National Research Fund (project: OTKA T034212) and that of the Hungarian-Portuguese S&T Project No. Port-6/03.

VIII. REFERENCES

[1] J.K. Tar, J.F. Bitó, K. Kozłowski, B. Pátkai, D. Tikk, "Convergence Properties of the Modified Renormalization Algorithm Based Adaptive Control Supported by Ancillary Methods", in *Proceedings of the 3rd International Workshop on Robot Motion and Control (ROMOCO '02)*, Bukowy Dworek, Poland, 9-11 November, 2002, pp. 51-56, ISBN 83-7143-429-4, IEEE Catalog Number. 02EX616.

[2] József K. Tar, Imre J. Rudas, János F. Bitó, Karel Jezernik, "A Generalized Lorentz Group-Based Adaptive Control for DC Drives Driving Mechanical Components", in *Proceedings of The 10th International Conference on Advanced Robotics 2001 (ICAR 2001)*, August 22-25, 2001, Hotel Mercure Buda, Budapest, Hungary, pp. 299-305, (ISBN. 963 7154 05 1).

[3] J.K. Tar, I.J. Rudas, Touré Balladji, "Minimum Operation Algebraic Adaptive Control for Robot Based on a Novel Branch of Soft Computing", in the *Proc. of EUREL '2000, Vol. 2, Session 3, (European Advancded Robotics Systems - Masterclass and Conference - Robotics 2000)*, 12-14 April 2000, The University of Salford, Manchester, UK.

[4] V.I. Arnold, *Mathematical Methods of Classical Mechanics* (in Hungarian), Műszaki Könyvkiadó Budapest, Hungary 1985.

[5] J.K. Tar, I.J. Rudas, J.F. Bitó, "Group Theoretical Approach in Using Canonical Transformations and Symplectic Geometry in the Control of Approximately Modeled Mechanical Systems Interacting with Unmodelled Environment", *Robotica*, Vol. 15, pp. 163-179, 1997.

[6] E. Miletics, G. Molnárka, "Taylor Series Method with Numerical Derivatives for Initial Value Problems", *Journal of Computational Methods in Sciences and Engineering*, Vol. 3, no. 3, pp. 319-329, 2003.

[7] E. Miletics, "Energy Conservative Algorithm for Numerical Solution of Initial Value Problems of Hamiltonian Systems", in *Proc.of the ICCC 2003*, Siófok, Hungary, CD issue, ISBN 9637154183, 1-4.

[8] A. Gemant. A Method of Analyzing Experimental Results Obtained from Elasto-Viscous Bodies. *Physics*, 7, pages 311-317 (1936).

[9] P. J. Torvik and R. L. Bagley, On the Appearance of the Fractional Derivative in the Behaviour of Real Materials, *ASME Journal of Applied Mechanics*, Vol. 51, pages 294-298, June 1984.

[10] C. G. Koh and J. M. Kelly, "Application of Fractional Derivatives to Seismic Analysis of Base-isolated Models", *Earthquake Engineering and Structural Dynamics*, Vol. 19, pages 229-241, 1990.

[11] J. Machado, A. Azenha, "Fractional-order Hybrid Control of Robot Manipulators", in *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, pages 788-793, 1998.

[12] J. Liouville: "Mémorie sur le calcul des différentielles a indices quelconcues", *J. Ecole Polytechn*, 13, 71-162, (1832).

[13] A. K. Grünwald, "Über 'begrenzte' Derivationen und deren Anwendung", *Zeitshrift für angewandte Mathematik und Physik*, 12, 441-480 (1867).

[14] K. B. Oldham and J. Spanier: "The Fractional Calculus: Theory and Application of Differentiation and Integration to Arbitrary Order. Academic Press, 1974.

[15] Yahya El Hini: "Comparison of the Application of the Symplectic and the Partially Stretched Orthogonal Transformations in a New Branch of Adaptive Control for Mechanical Devices", in *Proc. of the 10th International Conference on Advanced Robotics*, August 22-25, Budapest, Hungary, pp. 701-706, ISBN 963 7154 05 1.