High–Gain Control of Systems with Arbitrary Relative Degree: Speed Control for a Two Mass Flexible Servo System

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Abstract— **In this paper we present an adaptive control strategy for the speed control of a two–mass flexible servo system with inertia, spring and damping constants assumed to be unknown. The control strategy is based on the high–gain concept – to apply it to systems with higher relative degree, an adaptive PI–Controller is combined with a state–feedback loop. The goal is to track a given reference signal whereby a unknown but piecewise constant disturbance is present.**

I. INTRODUCTION

In the traditional control theory the plant is assumed to be perfectly known. On this basis an appropriate controller can be developed. But this is a very hard and unrealistic assumption, because in a predominantly portion of practical control tasks in industry the designer only has a rough knowledge about the plant to be controlled. Especially in the domain of mechatronic systems control strategies were investigated, that employ an identification unit to get more information about the plant under control. Mainly different types of neural networks are used for identification. But this approach possesses two main drawbacks: firstly convergence of the parameters is guaranteed only if the error–signal is persistently exiting. Secondly it takes rather long time for the parameters to converge.

In this paper we investigate a simple adaptive control scheme that does not need a model of the plant. For this reason an identification algorithm is dispensable.

The considered plant is a two–mass flexible servo system (TMS). This is a common example for electrical drive systems with an flexible shaft between the machine and the load. It is described in continuous–time state-space form [5]:

$$\begin{aligned} \dot{x} &= A\,x + b\,u + b_l\,l \\ y &= [1,0,0]\,x \end{aligned}$$

with:

$$A = \begin{bmatrix} -d/J_A & c/J_A & d/J_A \\ -1 & 0 & 1 \\ d/J_M & -c/J_M & -d/J_M \end{bmatrix} \qquad (1)$$

$$b = \begin{bmatrix} 0 \\ 0 \\ 1/J_M \end{bmatrix}$$

$$b_l = \begin{bmatrix} -1/J_A \\ 0 \\ 0 \end{bmatrix}$$

The torque M_M of the electrical drive with the limiting $|M_M| \leq 23Nm$ is considered as the input u, the load l is considered as the unknown but constant disturbance M_W. The output y is the velocity of the load ω_A. The state vector $x := [\omega_A, \Delta\varphi, \omega_M]^T$ contains the velocity of the drive, the angle of twist between the drive and the load and the velocity of the load. The dynamics of the TMS is determined by the following unknown physical parameters:

J_M: moment of inertia of the electrical drive $[kgm^2]$
J_A: moment of inertia of the load $[kgm^2]$
d: damping coefficient of the elastic shaft $[Nms/rad]$
c: stiffness of the elastic shaft $[Nm/rad]$

II. STATEMENT OF THE PROBLEM

For the unknown TMS introduced in Section I an appropriate control scheme has to be developed. To avoid the drawbacks of identification algorithms, a non-identifier based high–gain control strategy is used. This kind of controller is known to be globally asymptotically stable for systems of relative degree one. For the TMS having relative degree two[1] a suitable extension of the original controller will be necessary. The goal of control is to attain a given setpoint even in the presence of a unknown but constant disturbance acting on the plant. This may happen due to friction or a load–torque, induced by the work machine.

III. CONTROLLER STRUCTURE

The controller structure presented in this paper consists of two parts. In the feedforward path the adaptive PI–controller presented in [3] is used – this high–gain controller works with minimum-phase systems with relative degree one only. So the following assumptions can be stated:

Assumption (A1): The controlled system is relative degree one and minimum phase.

Assumption (A2): The unknown disturbance l is piecewise constant.

Since the TMS is of relative degree two, (A1) is violated

[1] Here the current control loop is assumed as fast enough and is thus neglected. So the torque M_M can be an arbitrary function of time. The inclusion of the current control loop increases the relative degree of the TMS and therefore makes the problem more complex. But simulations indicate that local stability is retained at least.

and global stability cannot be achieved by purely applying this PI-controller. The basic idea to fit the constraint of the low relative degree is to combine the PI-controller with a state feedback structure. The state feedback signal

$$y' = K\,x \qquad \text{with:} \qquad K = [K_0, K_1, K_2] \tag{2}$$

is interpreted as an artificial new output. The resulting transfer function $F'(s) = y'(s)/u(s)$ can easily be made relative degree one as well as minimum-phase, whereby the prerequisites are fulfilled. In this case global stability is achieved by the controller in [3], where a detailed proof can be found. This is true even in the case of constant acting disturbances. Furthermore a vanishing control error is achievable although the plant is unknown, so the desired setpoint is attained asymptotically. The adaptive PI–controller is given by

$$u(t) = k_1(t)\,e(t) + \int_0^t k_2(\tau)\,e(\tau)d\tau \tag{3}$$

with the control error $e = w - y' = w - Kx$. The time-varying PI–controller contains $k_1(t)$ as its proportional gain and $k_2(t)$ as its integral gain. These gains are adjusted by the adaptation laws

$$k_1(t) = e^2(t) + \alpha_1 \int_0^t e^2(\tau)d\tau \tag{4}$$

$$k_2(t) = \alpha_2 \int_0^t e^2(\tau)d\tau \tag{5}$$

with free design parameters α_1 and α_2. The integral term on the right hand side of the adaptation law (5) reveals the high-gain property of this controller type: the derivative $\dot{k}_2 = \alpha_2\,e^2 \geq 0$ clearly shows the monotony of the function $k_2(t)$. For applying state feedback, the states are assumed measurable which is true for our TMS. In the more general case of unmeasurable states a high–gain observer can be used [1], [2], [4].

For the purpose of decoupling the minimum–phase property from the physical parameters, the TMS is assumed to have controllable canonical form (CCF):

$$\begin{aligned} \dot{\xi} &= A_{CCF}\,\xi + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + b_{lCCF}\,l \\ y &= [\frac{c}{J_M J_A}, \frac{d}{J_M J_A}, 0]\,\xi \end{aligned}$$

with:

$$\begin{aligned} A_{CCF} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{J_M+J_A}{J_M J_A}c & -\frac{J_M+J_A}{J_M J_A}d \end{bmatrix} \\ b_{lCCF} &= J_M \begin{bmatrix} -1/c \\ 0 \\ 1/J_A \end{bmatrix} \end{aligned} \tag{6}$$

The feedback signal is now generated by $y' = K\xi$. Then it is a well-known fact that the feedback coefficients $K = [K_0, K_1, K_2]$ coincide with the coefficients of the numerator polynomial of the transfer function $F'(s)$. In other words the transmission zeros from $u(s)$ to $y'(s)$ are determined by the vector K only, which is a free design parameter. Consequently the transmission zeros can be placed arbitrarily without knowledge of the real system parameters. Therefore the minimum-phase assumption can be satisfied by an appropriate choice of K, i.e. the polynomial $z(s) = K_2\,s^2 + K_1\,s + K_0$ has to be strictly hurwitz. This means

$$K_i > 0 \qquad \forall i \in [0, 1, 2]. \tag{7}$$

The requirement of relative degree one is easily satisfied by the choice $K_2 \neq 0$. Yet obviously this is implied by the hurwitz conditions and thus adds no further restriction. The extension of the PI–controller by state feedback yields a control strategy that is suitable to control a integrator chain (CCF) with unknown parameters, affected by a piecewise constant but unknown disturbance.

IV. TRANSFORMATION ON CCF

Since the TMS was considered as a CCF in the previous section, the transmission zeros of $F'(s)$ are independent of the system parameters. This very helpful attribute is the reason why the minimum-phase condition can easily be met. But the physical TMS itself is not in CCF, cf (1). It is well-known that the input-output behavior of the physical realization (1) and the mathematical statespace model (6) coincide, but the real states x differ from the transformed states ξ of the model. Due to controllability we know about the existence of a similarity transformation, that converts the physical states into the required states of the CCF-model. If the state feedback was designed for the CCF-model, the measured states x of the TMS have to be transformed, if the controller should be applied to the real TMS. Because the parameters of the TMS are assumed to be unknown, the eigenvalues and the eigenvectors are unknown as well. But the required transformation matrix depends on the unknown eigenvalues and eigenvectors, so the conversion between the two different coordinate systems fails. In other words the measured states are useless for feedback in this case, while transformed states are needed. To overcome this obstacle two different solutions are investigated.

A. General Solution with an Observer Structure

To solve this problem in a general way, a non-adaptive high-gain observer (HGO) is designed [1], [2], [4]. In contrast to a Luenberger-observer a copy of the system cannot be used, because the observed system is unknown. The HGO consists of a integrator chain of the same order $n = 3$ of the TMS with high-gain feedback of the estimation error $e_x = x - \hat{x}$.

$$\begin{aligned} \dot{\hat{x}} &= A_{obs}\,\hat{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{b_{obs}} u + h\,c_{obs}\,e_x \\ \hat{y} &= \underbrace{[1, 0, 0]}_{c_{obs}}\,\hat{x} \end{aligned}$$

with:

$$A_{obs} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

$$h = \begin{bmatrix} \beta_1/\varepsilon \\ \beta_2/\varepsilon^2 \\ \beta_3/\varepsilon^3 \end{bmatrix}$$

This choice gives an error equation [2], [4]

$$\dot{e}_x = (A_{obs} - hc_{obs})\, e_x + b_{obs}\, \Delta(x, e_x) \qquad (9)$$

that requires the matrix $(A_{obs} - hc_{obs})$ to be hurwitz to show stability of the observer. The input $\Delta(x, e_x)$ of the error differential equation (9) is identically zero if a Luenberger-observer would be used. Because the TMS-parameters are unknown, this is not possible and the term $\Delta(x, e_x)$ cannot vanish. So this term is interpreted as a disturbance acting on the observer dynamics. For this reason the matrix $(A_{obs} - hc_{obs})$ has to be chosen such that a satisfactory attenuation of the perturbation $\Delta(x, e_x)$ is ensured. If the error gains β_i are chosen such that the polynomial

$$\beta = s^3 + \beta_1\, s^2 + \beta_2\, s + \beta_3 \qquad (10)$$

is hurwitz, then the eigenvalues of $(A_{obs} - hc_{obs})$ are $1/\varepsilon$ times the roots of (10). With decreasing ε the observer becomes faster and approximates an nth order differentiator. This would be an ideal observer for the required transformed states ξ, because the system is modelled as a integrator chain with the states $\hat{x}$ corresponding to the derivatives of y. With such an HGO that does not need any knowledge of the system the states of the CCF-model can be approximately reproduced. As usual the observer states $\hat{x}$ are fed back instead of ξ. The free parameter ε influences the dynamics of the observer and has to be chosen in a trade–off. On the one hand the observer has to be faster than the unknown dynamics of the TMS and therefore a small value seems suitable. But on the other hand for $\varepsilon \longrightarrow 0$ the observer converges to a differentiator and measurement noise will perturb the estimated values.

B. Special Solution with exploitation of the TMS-structure

The physical realization of the TMS contains an integrator chain but is not in CCF exactly. But its similar structure leads to a helpful attribute of the transfer function $F'(s)$ nevertheless. To be precise, it is not required that the transmission zeros itself are independent of the TMS-parameters, but the minimum-phase property has to be guaranteed, irrespective of the exact values of the TMS-parameters. So a straight forward calculation yields the transfer function

$$F'(s) = \frac{y'(s)}{M_M(s)} = \frac{y'(s)}{u(s)} = \qquad (11)$$

$$= \frac{K_2 J_A s^2 + (d(K_0 + K_2) + K_1 J_A)s + c(K_0 + K_2)}{J_A J_M s^3 + (J_A + J_M)ds^2 + c(J_A + J_M)s}$$

if the measurable states x are used for feedback. Indeed the numerator polynomial of (11) itself is not independent of the TMS but the minimum-phase condition is met if all its coefficients are strictly positive. This is ensured by the following conditions on K_i, because all TMS-parameters are assumed to be positive for physical reasons.

$$\begin{aligned} K_2 &> 0 \\ K_1 &> \underbrace{-d/J_A(K_0 + K_2)}_{<0} \\ K_0 &> -K_2 \end{aligned} \qquad (12)$$

The feedback coefficient K_1 has to be larger than some unknown negative constant, so for practical use we restrict it to non–negative values $K_1 \geq 0$ to ensure condition (12). With these requirements on the K_i which differ from (7) the minimum-phase condition is satisfied as well as the relative–degree–one condition. So the measurable states x can be used for feedback directly and no observer is needed. But this is a special case and is no longer true for arbitrary systems. In the general case the minimum-phase property depends on the unknown system parameters, especially if the parameters can also be negative for physical reasons.

V. SIMULATION RESULTS

For simulation studies the following data of the TMS is used:

J_M	=	0.166	$[kgm^2]$
J_A	=	0.333	$[kgm^2]$
d	=	10	$[Nms/rad]$
k	=	1220	$[Nm/rad]$

To achieve more realistic simulations the smooth function $M_R = 0.64 \arctan(10\omega_A)$Nm is implemented to approximate the friction. By this choice the disturbance l is not constant and therefore (A2) is violated. But for practical reasons (A2) can be considered as satisfied because of the saturation–like behavior of the arctan-function. A rather slow speed ω_A will exceed the steep slope of the function M_R and causes M_R to approximately adopt its maximum value. Additionally a load $M_L = 10\sigma(t - 2s)$Nm is assumed, with $\sigma(t)$ being the step-function.

$$\sigma(t) = \begin{cases} 0 & \forall \quad t < 0 \\ 1 & \forall \quad t \geq 0 \end{cases} \qquad (13)$$

The resulting load torque is given by $M_W = M_R + M_L$. The state feedback coefficients are set to $K_0 = 5$, $K_1 = 0$ and $K_2 = 10$. An optimization strategy for an applicable choice for the ratio between these coefficients is not available at present. Note, that condition (12) guarantees a stable control loop but nothing else is said about the transient behavior. For the weighting of the P and the I component we chose $\alpha_1 = 5$ and $\alpha_2 = 20$. The desired value function $w(t)$ is a step function filtered by a low pass of degree one. This enables the TMS to follow approximately. Here the desired value function is chosen as $w(t) = (K_0 + K_2) \cdot 2\pi(1 - \exp^{-\frac{t}{0.2s}})$. The reason for using the pregain $(K_0 + K_2)$ will be discussed in the next paragraph. The simulation result is shown in Fig. 1 to 3. The output ω_A converges to a constant value nearly as fast as the desired value does. No overshoot is recognizable. At $t = 2s$ the additional load torque is applied. This is shown in Fig. 2 and may be due to a closing clutch. The output

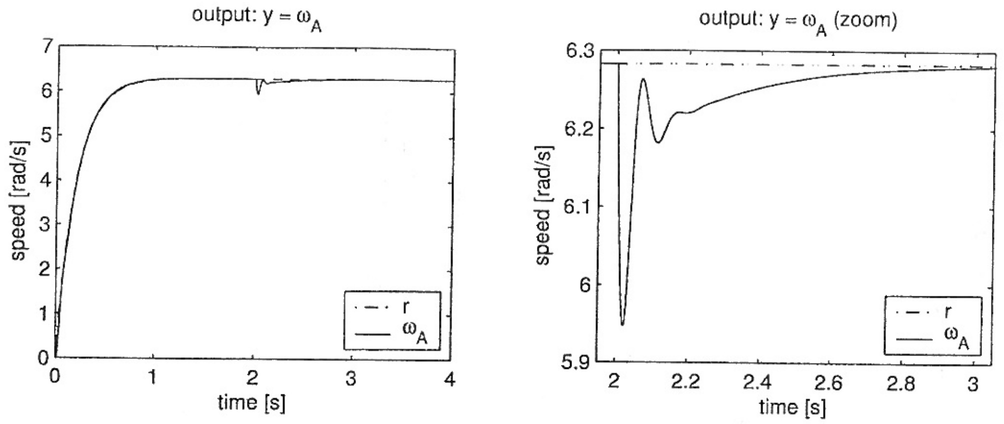

Fig. 1. Simulation result for the TMS without measurement noise. The output ω_A attains the setpoint $r = 2\pi$. The additional load torque applied at $t = 2$s reduces the output for a short instant but this effect can be compensated by the controller. The figure on the right hand side zooms the instant when the load is applied.

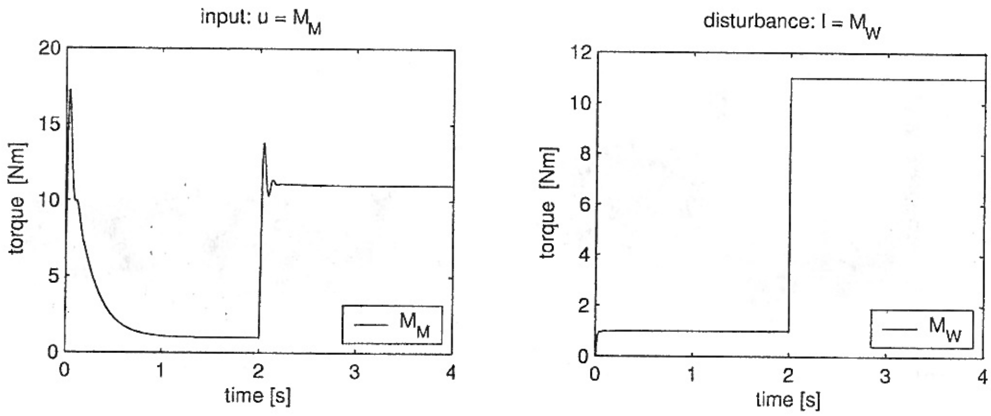

Fig. 2. Simulation result for the TMS without measurement noise. The left hand figure shows the input u which coincides with the torque of the drive. The limiting $|M_M| \leq 23$Nm is not exceeded. In the plot on the right hand side the disturbance is shown. In the interval $0 \leq t < 2$s only the friction $M_R = 0.64 \arctan(10\omega_A)$Nm is active. From $t \geq 2$s the additional constant load torque of 10Nm is applied.

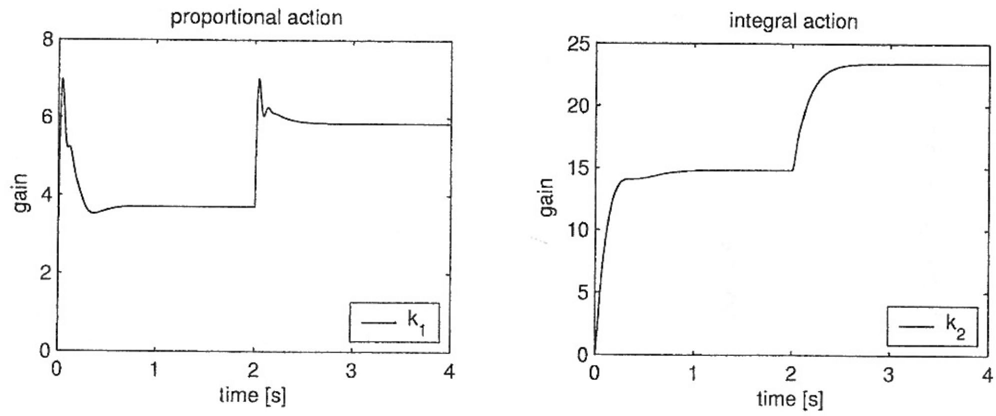

Fig. 3. Simulation result for the TMS without measurement noise. The parameters k_1 and k_2 are plotted. Both of them converge to a finite limit.

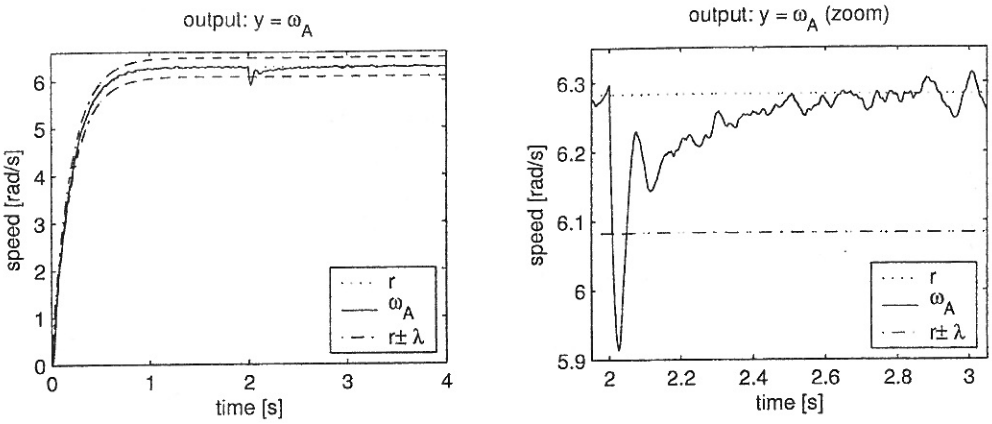

Fig. 4. Simulation result for the TMS with measurement noise $\nu(t)$. As in the noise-free case the output approaches the desired value $r = 2\pi$, but due to the noise y moves around r. So the error never converges to zero. The additional load reduces the output, this effect is compensated by the controller. Again, on the right hand side plot this incident is shown in a zoom. In both plots the λ−band is drawn in.

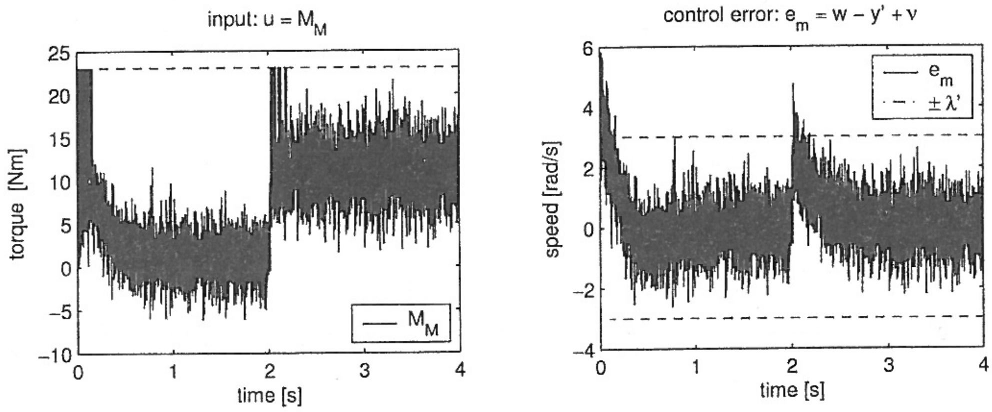

Fig. 5. Simulation result for the TMS with measurement noise $\nu(t)$. The input u is bounded to the saturation limit $|M_M| \leq 23\text{Nm}$. As a result of the sudden load torque the error signal leaves the predefined λ−band. In contrast to this, the noise does not force the error signal to leave the band.

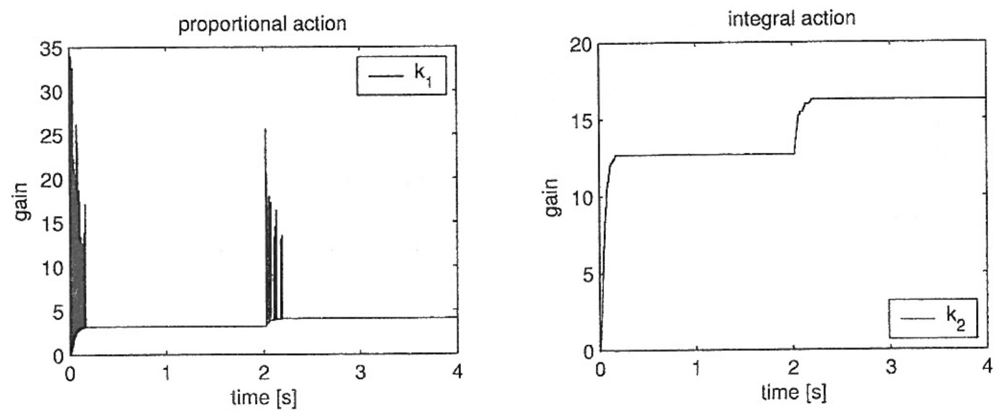

Fig. 6. Simulation result for the TMS with measurement noise $\nu(t)$. Despite the noise the gains converge. Only the load torque causes a short period of adaptation, when the λ−band is left.

decreases a little but the controller copes with this constant disturbance.

The gains k_1 and k_2 are increased (see Fig. 3) by which the output is forced back to its previous value. A more detailed zoom of this procedure is shown in the right hand plot of Fig. 1. As the simulation shows the high-gain parameter k_2 converges as well as k_1 does. The input M_M stays within the physically given limit of $\pm 23Nm$. The control error $e = w - y'$ vanishes, forced by the integral action of the PI–controller.

The goal of control is the "real" output y – not the artificial output y' – to track the given reference signal. But here w is given as the set point value for y'. Note that in the steady-state the ratio y'/y can be made independent of the TMS. In the steady-state we have:

$$\begin{aligned} \omega_A &= \omega_M && (14) \\ \Delta\varphi &= M_W/c && (15) \\ M_M &= M_W = c\,\Delta\varphi && (16) \end{aligned}$$

The feedback signal y' is defined by

$$y' = K_0 x_1 + K_1 x_2 + K_2 x_3 = K_0 \omega_A + K_1 \Delta\varphi + K_2 \omega_M \quad (17)$$

and further it is known that $y = x_1 = \omega_A = \omega_M$ is valid in the steady-state case. So a relation between y and y' follows:

$$y' = (K_0 + K_2)y + \frac{K_1}{c} M_W \quad (18)$$

To become independent of the unknown c and M_W, we have set $K_1 = 0$ which is admissible, cf (12). So the steady state control accuracy no longer depends on the TMS-parameters. An additional pre-gain $K_0 + K_2$ adjusts the setpoint value for y'. For that reason w is given by

$$w = (K_0 + K_2)\,r \quad (19)$$

where r is the desired value for y. This constant pregain was implemented for all simulation studies.

In practical applications controllers have to deal with measurement noise $\nu(t)$. The measured control error e_m will be given by $e_m = e + \nu = w - y' + \nu$. In the adaptation law (5) e gets substituted by e_m now, so measurement noise will prevent k_1 and k_2 converging even if $e = w - y'$ is identically zero. For this reason an extension will be implemented, in the literature known as λ-tracking. The error e_m is set to zero if its absolute value is less than a predefined limit λ':

$$e_m = \begin{cases} e + \nu & \text{if} \quad |e_m| > \lambda' \\ 0 & \text{if} \quad |e_m| \le \lambda' \end{cases} \quad (20)$$

This modification is applied to the error signal in (4) and (5) only. Note that in (3) the measured error $w - y' + \nu$ is always used – this error is never set to zero. For this reason the control action u reacts on every noise signal, even when the measured error signal is less then λ'. Simply the adaptation is disabled for $|e_m| \le \lambda'$, so the gains k_1 and k_2 remain on constant values in this case. To relate the value of λ' to the desired value r the same gain as in (19) is used to calculate λ:

$$\lambda' = (K_0 + K_2)\,\lambda \quad (21)$$

In Fig 4 to Fig. 6 the simulation result is shown when measurement noise ν is applied. The limit λ was set to 0.2 rad/s. Basically the result is the same as without noise. The gain coefficients k_1 and k_2 converge. The noise-disturbed error e_m results in high frequency components in the input M_M. As a matter of fact the TMS acts as a low pass filter by what the high frequency components are damped in the output signal.

VI. Conclusion

In the presented paper, a two–mass flexible servo system with unknown physical parameters was controlled by an adaptive control scheme. The applied controller neither identifies the system parameters nor requires a model of the plant under control. Only the degree of the plant is assumed to be known and the states have to be measurable. The latter assumption may be relaxed if a high–gain observer is implemented.

Despite the fact that the TMS is of relative degree two, a high–gain feedback control can be applied to achieve global stability. Furthermore the presented controller structure is generalized by a high–gain observer. This enables the application of high–gain feedback control to systems with arbitrary relative degree and the achievement of global stability. The observer incorporates some extra dynamics in the feedback path and can be interpreted as a kind of filtering of the output signal. As shown in section IV-A the observer approximates a differentiator of the order of the plant.

The presented controller structure also copes with unknown process parameters and deals with unknown disturbances, so that the control error converges to zero. Even in the case of noisy measurement signals this goal is achieved.

The problem of selecting convenient feedback coefficients remains unsolved. For time-varying systems the monotony of the gain is expected to cause difficulties. These two problems will be topics of future investigations.

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