CAD OF OPTIMAL CONTROL OF THE HEAT DISSIPATION IN SEMI-INFINITE SPACE

Dr. Eng. DANIELA CÂRSTEA [1],
Dr. Eng. ION CÂRSTEA [2],
[1] Industrial Group CFR, Craiova,E_mail:ncarstea@hotmail.com
[2] University of Craiova, 1100 Craiova, ROMANIA

Abstract:- The paper presents the optimal control problem of the heat penetration in semi-nfinite space, using boundary commands like the boundary temperature (Dirichlet's command) . The necessary conditions for optimality are obtained by a variational approach. The gradient technique is used to determine the optimal command. A numerical model is developed using the co-state equations. We developed some algorithms for a large class of problems in this area. Some examples illustrate the use of our product in the heat penetration in conductors.

Keywords: Heat transfer, inverse problems, optimisation, CAD.

I. INTRODUCTION

The design of any physical system must fit some requirements, which constitute the technical specifications (the constraints). In the frame of these constraints some performances must be optimised in order to build the best system. The scope of this paper covers a large class of engineering applications in the field of the heat transfer in conductors.

An elliptical equation (steady-state problem) or a parabolic equation (unsteady-state problem) can describe the heat transfer by conduction. In the heat transfer in electrical devices two aspects of the problem appear:

- An analysis of the temperature distribution with imposed boundary conditions (specified temperature, convective and radiation flow).
- Optimal control of the heat transfer, either distributed or boundary commands.

The first aspect is treated in most works and consists in determining of the temperature distribution in the parts of the system when the geometry of the system and the thermal load are known (that is the internal heat sources and the boundary conditions are given). The second aspect is more complex because it requires controlling the heat transfer that is to determine the values of certain variables called the commands so that the system has a desired evolution. More, we seek those commands, called optimal commands, that lead to the best evolution with respect a known criterion.

The boundary commands are easy implemented because the boundary is an interface between environment and the controlled system, although the advanced technologies permit implementation of the distributed commands.

II. PROBLEM FORMULATION

We limit our discussion at one-dimensional case. The problem dealt with in this paper is governed by the following differential equation:

$$\frac{\partial \theta}{\partial t} = K\frac{\partial^2 \theta}{\partial x^2} \tag{1}$$

defined for a time interval $T=[0,t_f]$ over a one-dimensional domain $[0,x_f]$. $\theta(x, t)$ is the state function (the temperature in our example) in the point x at the time t, and K a material constant. System control is accomplished via boundary controls.

We seek to minimise:

$$J(u) = c0 \int_0^{x_f} [\theta(x,t_f) - \theta_D(x)]^2 dx \tag{2}$$

by choosing an optimum control u^* from a set of admissible functions U, such that

$$J(u^*) \le J(u), \quad \forall u \in U$$

θ_D is a desired temperature distribution at the final time of the control interval T, and c0 is a positive weighted constant. The functional cost has a practical significance: it penalises the deviations of the temperature in the domain from the imposed standard. The objective is to obtain the temperature profile in the interior region of the conductor at certain prescribed values, while keeping a constant heat flux on the conductor.

The idealised problem investigated consists in a copper conductor placed under influence of a constant temperature V on the boundary x=0. The conductor is assumed to be infinitely long in the x direction with the boundary the plane Oyz (see figure 1). Let us suppose that for t<0, the solid temperature is 0 and a step temperature V is applied at the moment t=0.

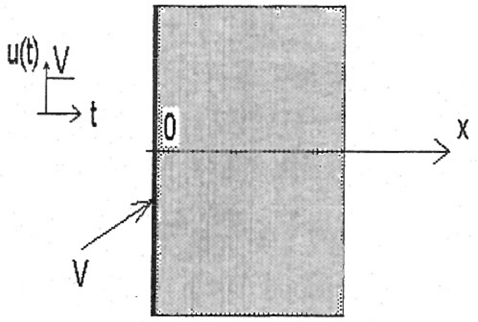
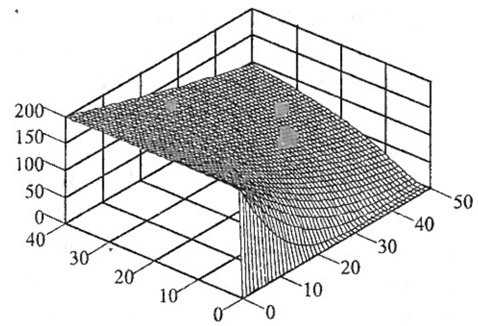


Figure 1. The domain



M

Figure 2. The temperature distribution

In an analysis problem, the temperature θ(x, t) along the axis Ox is required.

The initial condition is:

$$\theta(0,x)=0$$

The boundary and limit conditions are:

$$\theta(x,0)=0; \quad ;\theta(0,t)=u(t)$$
$$\lim_{x\to\infty}\theta(x,t)=0 \tag{3}$$

The command u (t) is defined as Heaviside function:

$$u(t)=\begin{cases}V,\forall t\geq 0\\ 0,\forall t\prec 0\end{cases}$$

The analytical solution for temperature is [2, 6]:

$$\theta(x,t)=V[1-erf(\frac{x}{2.\sqrt{K.t}})] \tag{4}$$

with λ the thermal conductivity of the material (copper in our case). The function erf (x) is the error function of Gauss.

The command is the temperature V on the boundary of the analysis domain (plane defined by x=0). In the figure 2 the temperature distribution is represented for a given value of the step using the program MathCad [4]. As we see, with time increasing the temperature on the boundary increases and the penetration depth increases too.

The value for the K is determined with the expression:

$$K=\frac{\lambda}{c.\gamma}$$

with c - specific heat and γ the specific mass.

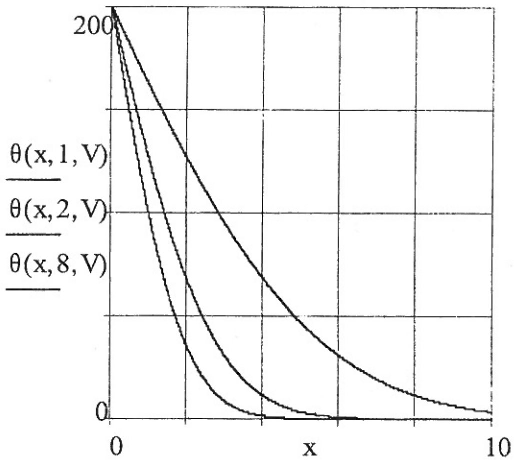


Figure 3. Penetration depth in time (x_f=10)

The figure 3 represents the depth penetration for different time intervals. We considered a time interval [0, 8 s] and a length in the direction Ox equal to x_f=3 m. The step value is V=200. We considered a division of the time interval in 40 points and space interval in 50 points. The penetration depth depends on time, the boundary temperature and the conductor properties. In our examples we consider that the material properties are constant with the temperature variations.

A special case is for a command u(x,t)=V.δ(x).δ(t) (that is a Dirac impulse on the boundary plane). We have a plane source for a short time.

The solution of the heat equation is:

$$\theta(x,t)=V.\sqrt{\frac{1}{4\pi Kt}}e^{\frac{-x^2}{4Kt}}.1(t)$$

with 1(t) – Heaviside function.

In figure 4 the heat distribution is plotted for an interval of 0.002 m divided in 50 points and a time interval [0, 2.10^{-6} s] divided in 80 points. The amplitude of the Dirac impulse is V=550 ^{0}C. The heat penetration for two different times is presented in the figure 5.

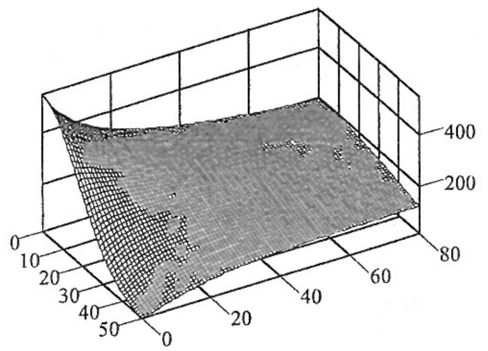


Figure 4. The variation of temperature

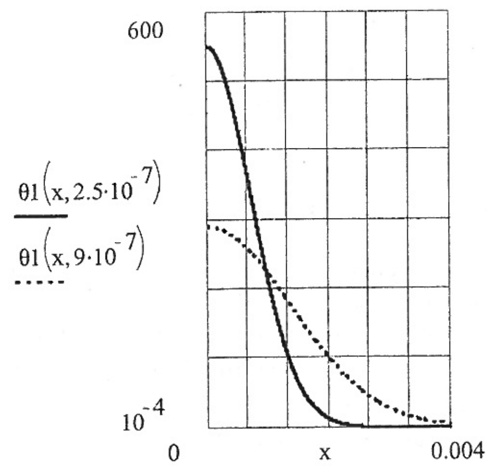


Figure 5. Heat penetration for different times

III. NECESSARY CONDITIONS FOR OPTIMALITY

We transform the constrained optimal control problem into an unconstrained problem through the introduction of adjoint function Φ. We define the augmented cost-functional by [1]:

$$L = J(u) + \int_0^{tf}\int_0^{xf} \Phi(x,t)\left[\frac{\partial\theta}{\partial t} - K\frac{\partial^2\theta}{\partial x^2}\right]dx.dt$$

Necessary conditions for optimality are derived by a variational approach. It is considered a variation δf in the command f that introduces a variation δL. From the first variation of L, results the adjoint equation:

$$\frac{\partial\Phi}{\partial t} + K\frac{\partial^2\Phi}{\partial x^2} = 0 \tag{5}$$

with the limit conditions:

$$\Phi(tf,x) = 2.c_0[\theta(tf,x) - \theta_d(x)]$$

$$\Phi(0,t) = 0; \quad \Phi(xf,t) = 0$$

$$J'(u) = K.\frac{\partial\Phi(0,t)}{\partial x}$$

The optimal command u^*can be obtained by solving the state and co-state equations (1) and (5). A numerical model can be used as the finite element method. The gradient technique was used in some previous works of the authors with good results in 2D models.

IV. AN OPTIMISATION ALGORITHM

There are many deterministic methods capable of solving the optimisation problem defined above. Among the various methods, which can be used, the gradient methods based upon the evaluation of partial derivatives of the objective function seem the best way to look for an optimal value of the design parameters.

The gradient methods need the computation of the derivatives of various quantities. The computation of these derivatives is computer time consuming. It is thus useful to consider a way of optimisation without the derivative evaluation. To move towards the minimum of this object function, we must change the parameter of description of the system against the slope of the object function J (u); so that we move to the minimum:

$$u^{i+1} = u^i - s.\frac{\partial J^i}{\partial u} \tag{6}$$

where i is the iteration index and s is the step length in the anti-gradient direction.

The algorithm in pseudo-code has the following form [5]:

1. *Make an initial guess of the command u_0, and set the iterations counter to zero;*
2. *Compute the new command with(6);*
3. *Solve the equation (5);*
4. *Compute the performance index (2);*

5. *Repeat the steps $2^0 - 4^0$ until subsequent changes in J are less than a pre-set criterion.*

The length of the step s is determined by a one-dimensional search technique.

V. SOME APPLICATIONS

If the goal is to have a certain profile of the temperature at the final time, we can use the object function defined by (3). The final profile can be a non-linear or linear function.

Case 1: A non-linear profile for the temperature

The object function is to have a desired non-linear profile at the final time and an object function (2) with t_f=8, x_f = 10 and u_{min}=0 and u_{max}=250.

The object function is represented in figure 6 for c_0=0.001. In figure 7 the variation of the profile temperature with the command in the iterative process is plotted.

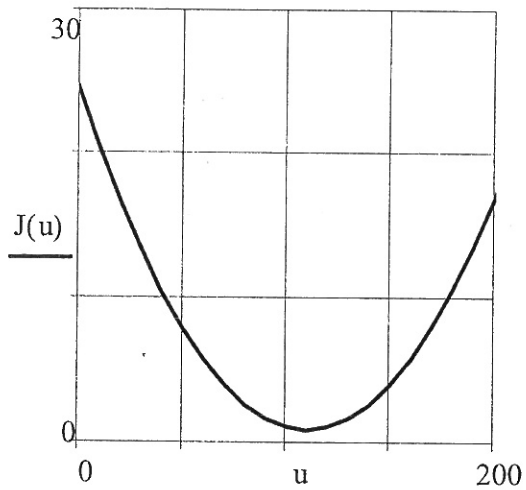


Figure 6. The object function and minimum

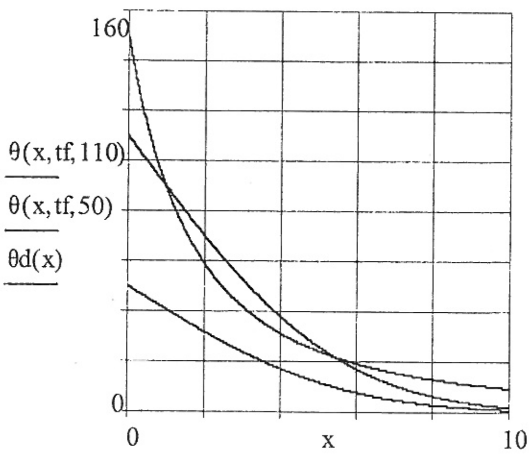


Figure 7. The temperature penetration

We consider the case of an imposed temperature profile by the form:

$$\theta_D = \frac{150}{(0.3x+1)^2}$$

Case 2: A linear profile for the temperature

We consider the case of a linear profile, that is:

$$\theta_D(x) = -15.x + 150$$

The parameters has the values t_f=8, x_f=10, u_{min}=100 and u_{max}=250. In figure 8 the variation of the J(u) is represented and the variation of the temperature profiles in the minimisation process is illustrated in figure 9. The optimal value for u is u^*=190.624 and the minimum for object function is 0.493 for c_0=0.0001. The iteration number is 13 with the start value for u equal to 190.

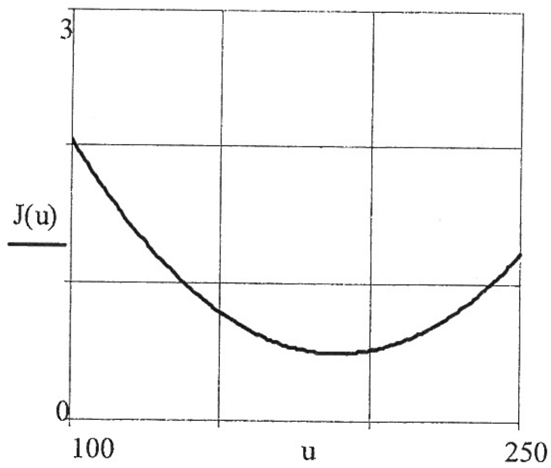
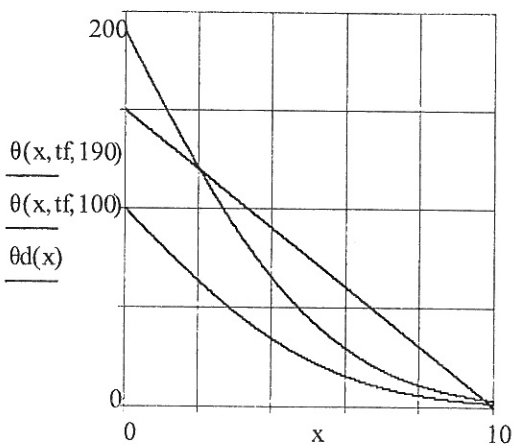


Figure 8. The object function for linear profile

Figure 9. The temperature profiles



VI.CONCLUSIONS

In this paper an algorithm for the optimal control of the heat penetration in solids is presented. We tried to illustrate the importance of the optimal control of the heat penetration in an actual engineering problem. Numerical results have been given for the optimal control of the heat penetration in one space variable but can be extended in a multi-dimensional space. In some previous works we presented this extension with finite element method.

VII. REFERENCES:

[1]. Cârstea, D. Numerical simulation of magneto-thermal processes using automatic computation systems. Ph.D. Thesis, 1999, University of Ploiesti, Romania.

[2]. Hortopan, G. Aparate electrice. EDP, Bucharest, 1967.