A control algorithm for non-linear processes using on-line simulation and rule based control

Radu Bălan
Department of Mecatronics
Technical University of Cluj-Napoca
B-dul Muncii 103-105 Cluj-Napoca
Romania
radubalan@yahoo.com

Vistrian Mătieş
Department of Mecatronics
Technical University of Cluj-Napoca
B-dul Muncii 103-105 Cluj-Napoca
Romania
matiesvistrian@yahoo.com

Olimpiu Hancu
Department of Mecatronics
Technical University of Cluj-Napoca
B-dul Muncii 103-105 Cluj-Napoca
Romania
ohancu@yahoo.com

Abstract – **Model predictive control (MPC) is an optimization-based approach that has been successfully applied to a wide variety of control problems. When MPC is employed on nonlinear processes, the application of this typical linear controller is limited to relatively small operating regions. The accuracy of the model has significant effect on the performance of the closed loop system. Hence, the capabilities of MPC will degrade as the operating level moves away from its original design level of operation. A solution to avoid these problems is multiple model adaptive control approach (MMAC) which uses a bank of models to capture the possible input-output behavior of processes. In most of these strategies, the controllers are based on linear models with fixed parameters so that the vast body of linear control theory can be applied. Other solutions include the use of a nonlinear analytical model, combinations of linear empirical models or some combination of both. This paper presents an MPC algorithm which uses on-line simulation and rule-based control. The basic idea is the on-line simulation of the future behaviour of control system, by using a few control sequences and based on nonlinear equations of the analytical model. Finally, the simulations are used to obtain the 'optimal' control signal. These issues will be discussed and nonlinear modeling and control of two processes will be presented as examples: a single-pass concentric-tube counter flow heat exchanger and the inverted pendulum on a cart.**

I. INTRODUCTION

Model Predictive Control (MPC) refers to a class of algorithms that utilize an explicit process model to compute the control signal by minimizing an objective function. The performance objective typically penalizes predicted future errors and manipulated variable movement subject to various constraints. The ideas appearing in greater or lesser degree in all the predictive control family are basically:

-explicit use of a model to predict the process output in the future;

-on line optimization of a cost objective function over a future horizon;

-receding strategy, so that at each instant, the horizon is displaced towards the future, which involves the application of the first control signal of the sequence calculated at each step.

Performance of MPC could become unacceptable due to a very inaccurate model, thus requiring a more accurate model. This task is an instance of closed-loop identification and adaptive control. Here it is important to remember that the model is only used as an instrument in creating the best combined performance of the controller and the actual system, so the model does not necessarily need to be a good open-loop model of the system. The performance measure should be able to capture as much of the closed loop behavior as possible.

Let's consider that it is possible to compute:

- the predictions of output over a finite horizon (N);
- the cost of an objective function,

for each possible sequence:

$$u(.) = \{u(t), u(t+1), .., u(t+N)\} \tag{1}$$

and then to choose the first element of the optimal control sequence. For a first look, the advantages of the proposed algorithm include the following:

-the minimum of objective function is global;

-it is not necessary to invert a matrix, so potential difficulties are avoided;

-it can be applied to nonlinear processes if a nonlinear model is available;

-the constraints (linear or nonlinear) can easily be implemented.

The drawback of this scheme is a very long computational time, because there are possibly a lot of sequences. For example, if u(t) is applied to the process using a "p" bits numerical-analog converter (DAC), the number of sequences is 2^{p*N}. Therefore, the number of sequences must be reduced.

II. CONTROL ALGORITHM

A model based adaptive-predictive algorithm which uses on line simulation and rule based control, designed for linear processes, is developed in [8]. This algorithm can be applied with some modifies to nonlinear processes.

The nonlinear equations of the process can be used directly in the control algorithm. The predictions of system output are calculated by integrating the nonlinear ordinary differential equations of the model over the prediction horizon, by using a few control sequences.

For a first stage, are used, the next four control sequences:

$$\begin{aligned} u_1(t) &= \{u_{\min}, u_{\min}, .., u_{\min}\} \\ u_2(t) &= \{u_{\max}, u_{\min}, .., u_{\min}\} \\ u_3(t) &= \{u_{\min}, u_{\max}, .., u_{\max}\} \\ u_4(t) &= \{u_{\max}, u_{\max}, .., u_{\max}\} \end{aligned} \tag{2}$$

where u_{min} and u_{max} are the limits of the control signal.

There are two pair sequences: ($u_1(t)$, $u_2(t)$) and ($u_3(t)$, $u_4(t)$) which are different through the preponderance of u_{min} or u_{max} in the future control signal. The pair sequences are different only through the first term.

In the second stage, depending by the behavior of control system, it is used an algorithm that modifies the limits of control signal:

$$u_{min} \le u_{minst}(t) \le u(t) \le u_{maxst}(t) \le u_{max} \quad (3)$$
$$\Delta u_{min} \le \Delta u \le \Delta u_{max} \quad (4)$$

In relations (2), the values of u_{max}, u_{min} are replaced with $u_{minst}(t)$, $u_{minst}(t)$. The control signal is computed using a set of rules based on the extremes (max_0, max_1, min_0, min_1) of the error of output predictions (e_i, $_{i=1..4}$ are predicted errors, d is dead time, t_0 is current time, t_1 is the horizon of output, δ is a parameter which is used for a fine-tuning (first, it is simpler to consider δ=0)):

Rule 1: If the sequence u_3(t) leads to:
$$\min{}_0 = \min_{t0+d<t<t1} \{e_3(t)\} \quad \min{}_0 > \delta \quad (5)$$
In this case $u(t)=u_{minst}(t)$.

Rule 2: If the sequence u_2(t) leads to:
$$\max{}_1 = \max_{t0+d<t<t1} \{e_2(t)\} \quad \max{}_1 < -\delta \quad (6)$$
In this case $u(t)=u_{maxst}(t)$.

Rule 3: If the sequence u_4(t) leads to:
$$\min{}_1 = \min_{t0+d<t<t1} \{e_4(t)\} \quad \min{}_1 < -\delta \quad (7)$$
and: $e_4(t_0+d+1)>0$.
In this case $u(t)=u_{maxst}(t)$.

Rule 4: If the sequence u_1(t) leads to:
$$\max{}_0 = \max_{t0+d<t<t1} \{e_1(t)\} \quad \max{}_0 > \delta \quad (8)$$
and: $e_1(t_0+d+1)<0$.
In this case $u(t)=u_{minst}(t)$.

Rule 5: In majority of the other situations, it is used u_2 and u_3 sequences to obtain the control signal using a linear relation:

$$u(t) = \frac{u_{\min st}(t)\max{}_1 - u_{\max st}(t)\min{}_0}{\max{}_1 - \min{}_0} \quad (9)$$

A good behaviour of the control algorithm leads to a prevalence of rule 5. Other rules are used to modify the values of u_{maxst}, u_{minst} and to stabilise the control signal.

In this form, this algorithm does not address processes where the gain of the process changes sign.

III. THE MODEL OF THE HEAT EXCHANGER

Heat exchangers are devices that facilitate heat transfer between two or more fluids at different temperatures. Usually, model predictive control (MPC) uses a linear model and an on-line least square algorithm (RLS) to determine the parameters. Heat exchangers are nonlinear processes. To apply the standard MPC algorithms it is possible to use multiple model adaptive control approach (MMAC) which uses a bank of models to capture the possible input-output behavior of processes [3]. Other solutions are based on neural networks and fuzzy logic [4], [5]. In this paper it is used an example from [6]: a heat exchanger with hot fluid -engine oil at 80°C, cold fluid - water at 20° C, by using a single-pass counter flow concentric-tube. Other data and notations: length (L): 60m, heat transfer coefficients (k_1=1000 W/(m^2 °C), k_2=80 W/(m^2 °C)), the temperature profile of fluids and wall ($\theta_1(z,t)$, $\theta_2(z,t)$, $\theta_W(z,t)$), specific heat (c_1, c_2, c_w), cross-sectional area for fluids flow and wall (S_1, S_2, S_w), density of fluids and wall (ρ_1, ρ_2, ρ_w), flow speed of fluids (v_1, v_2), transfer area (S) (fig. 1).

If physical properties (density, heat capacity, heat transfer coefficients, flow speed) are assumed constant, the heat exchanger model is described using a shell energy balance as:

-hot fluid:
$$c_1\rho_1 S_1 \frac{\partial\theta_1(z,t)}{\partial t} - c_1\rho_1 v_1 S_1 \frac{\partial\theta_1(z,t)}{\partial z} = \frac{k_1 S}{L}[\theta_w(z,t) - \theta_1(z,t)] \quad (10)$$

-cold fluid:
$$c_2\rho_2 S_2 \frac{\partial\theta_2(z,t)}{\partial t} + c_2\rho_2 v_2 S_2 \frac{\partial\theta_1(z,t)}{\partial z} = \frac{k_2 S}{L}[\theta_w(z,t) - \theta_2(z,t)] \quad (11)$$

-wall:
$$c_w\rho_w S_w \frac{\partial\theta_w(z,t)}{\partial t} = \frac{S}{L}[k_1\theta_1(z,t) + k_2\theta_2(z,t) - (k_1+k_2)\theta_w(z,t)] \quad (12)$$

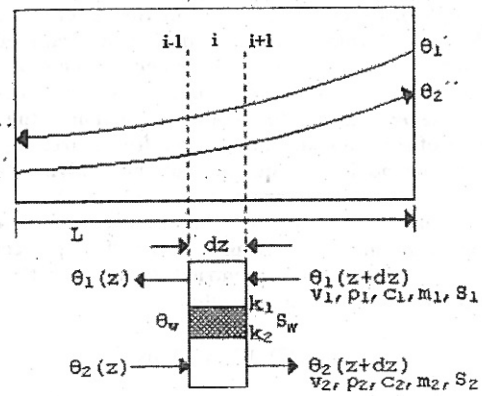

Fig. 1: Temperature distributions

Using general notation $\theta_{a(i,j)}$ with a=1 (hot fluid), a=2 (cold fluid), a=w (wall), i, j discrete elements in space respectively time, the discrete equations corresponding to partial differential equations (10),(11),(12) are:

$$\theta_1(i,j+1) = \theta_1(i,j)\left[1 - v_1\frac{\Delta t}{\Delta z} - \frac{k_1 S\Delta t}{Lc_1\rho_1 S_1}\right] + v_1\frac{\Delta t}{\Delta z}\theta_1(i+1,j) + \frac{k_1 S\Delta t}{Lc_1\rho_1 S_1}\theta_w(i,j) \quad (13)$$

$$\theta_2(i,j+1) = \theta_2(i,j)\left[1 + v_2\frac{\Delta t}{\Delta z} - \frac{k_2 S\Delta t}{Lc_2\rho_2 S_2}\right] - v_2\frac{\Delta t}{\Delta z}\theta_2(i+1,j) + \frac{k_2 S\Delta t}{Lc_2\rho_2 S_2}\theta_w(i,j) \quad (14)$$

$$\theta_w(i,j+1) = \theta_w(i,j) + \frac{S\Delta t}{L}[k_1\theta_1(i,j) + k_2\theta_2(i,j) + (k_1+k_2)\theta_w(i,j)] \quad (15)$$

In a control application, these equations can not be used directly because v_1 and v_2 are not constant in time. Let's consider next assumptions:

-the speed of fluids is limited: $v_{1(\min)}<v_1<v_{1(\max)}$; $v_{2(\min)}<v_2<v_{2(\max)}$; $v_{\max}=\max\left(v_{1(\max)}, v_{2(\max)}\right)$ (16)

- the fluids speed is only time-function:
$v_1=v_1(t)$, $dv_1/dz=0$, $v_2=v_2(t)$, $dv_2/dz=0$ (17)

- the length of heat exchanger is divided in n intervals:
$L=n\Delta z$; (18)

- in a time interval Δt, the fluids cover only a part of Δz:
$n_v v_{\max}\Delta t=\Delta z$; $\Delta t < L/(nn_v v_{\max})$ (19)

- two variables Δz_1, Δz_2 are using to totalize the small fluid displacements: $\Delta z_1(t+\Delta t)=\Delta z_1(t)+v_1\Delta t$;
$\Delta z_2(t+\Delta t)=\Delta z_2(t)+v_2\Delta t$ (20)

- in simulations, the displacements of the fluids become effective only if $\Delta z_1>\Delta z$ or/and $\Delta z_2>\Delta z$; in these cases:
$\Delta z_1\leftarrow \Delta z_1-\Delta z$ or/and $\Delta z_2\leftarrow \Delta z_2-\Delta z$ (21)

In other words, in simulations, the continue moves of fluids are replaced with small discrete displacements. As a result, the heat exchanger model is described by equations:

$$\theta_1(i,j+1)=\theta_1(i,j)\left[1-\frac{k_1 S\Delta t}{Lc_1\rho_1 S_1}\right]+\frac{k_1 S\Delta t}{Lc_1\rho_1 S_1}\theta_w(i,j) \quad (22)$$

$$\theta_2(i,j+1)=\theta_2(i,j)\left[1-\frac{k_2 S\Delta t}{Lc_2\rho_2 S_2}\right]+\frac{k_2 S\Delta t}{Lc_2\rho_2 S_2}\theta_w(i,j) \quad (23)$$

$$\theta_w(i,j+1)=\theta_w(i,j)+ + \frac{S\Delta t}{L}\left[k_1\theta_1(i,j)+k_2\theta_2(i,j)+(k_1+k_2)\theta_w(i,j)\right] \quad (24)$$

In a practical implementation, there are used equations (20), (21), (22), (23), (24).

It is important the number and position of temperature sensors. Here, it is considered that only the inlet and outlet temperatures (hot fluid, cold fluid, wall) and the flow rate of fluids are measured. The temperatures inside heat exchanger are estimated. The quality of heat exchange depends especially by the heat transfer coefficients. These parameters depend by temperatures, accumulation of deposits of one kind or another on heat transfer surface, shape of tube, etc. The temperature distributions inside heat exchanger (process and model) are presented in fig. 2 using notation $\theta_a(i,j)$. Analogous, the notation $M\theta_a(i,j)$ is used for the model.

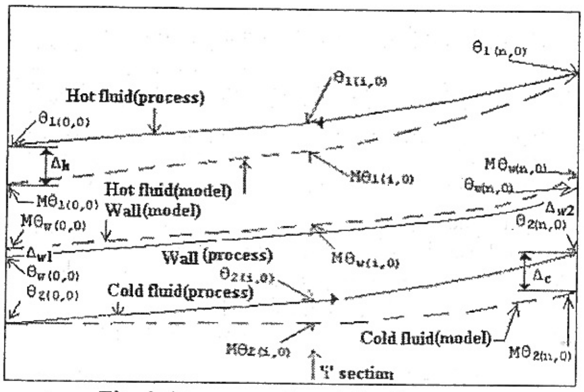

Fig. 2: Process and model - diagrams

At every sample period, it is possible to compute Δ_h, Δ_c, Δ_{w1}, Δ_{w2}, the temperature prediction errors of outlet hot fluid, outlet cold fluid, wall.

These predictions are used to correct the temperature distributions inside the model of heat exchanger, using translations and rotations of distributions. Also, prediction errors can be used to modify the parameters of the model using an algorithm based on rules.

IV. APPLICATIONS WITH HEAT EXCHANGER

The next applications show the main features of the algorithm applied to heat exchanger. The set point has a variable shape (42°C, 47°C, 52°C, 47°C, 42°C..). The limits of u(t) (hot fluid flow rate) are: $0.05\le u(t)\le 0.5$ [kg/s]. The flow rate of cold fluid is constant (0.08 kg/s). The temperatures of cold fluid (20°) and hot fluid (80°) are constant. Some experiments with variable flow rate or/and variable temperature of cold fluid are presented in [2].

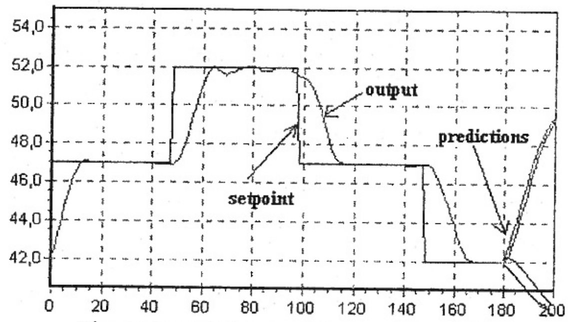

Fig. 3: Setpoint, output (accurate model)

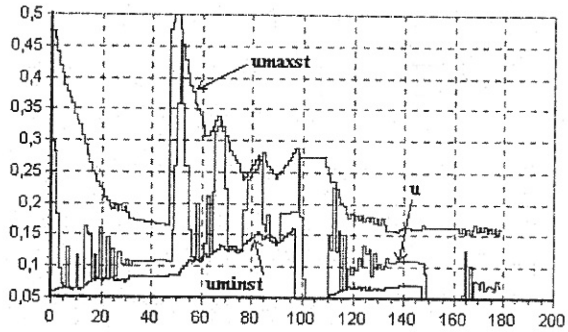

Fig.4:Controller output (accurate model)

First, it is used an accurate model (Fig. 3, fig. 4). If the algorithm uses only 1..5 rules, the variance of $u(t)$ will be large. To reduce this variance, a solution is to use a funnel zone for control signal, based on inequality (3). For example, if rule 5 is active then u_{maxst} decreases and u_{minst} increases.

Another solution is to limit Δu, using inequality (4). In steady-state regime, control signal is computed using average of past and new values. The algorithm do not uses directly an integral component. In figure 3, steps 50..80, the algorithm tries to reduce the error as fast as possible. As a result, a damped oscillation appears. To avoid this behavior, a solution is to use a reference trajectory.

In figure 5, 6 it is presented an adaptive case; the heat transfer corfficients depend by temperature:

$$k=k_0(1+\theta/200) \quad (25)$$

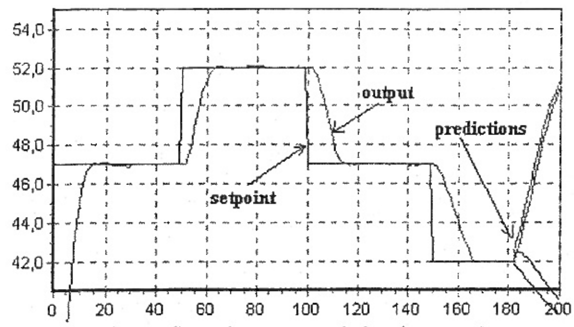

Fig. 5: Setpoint, output (adaptive case)

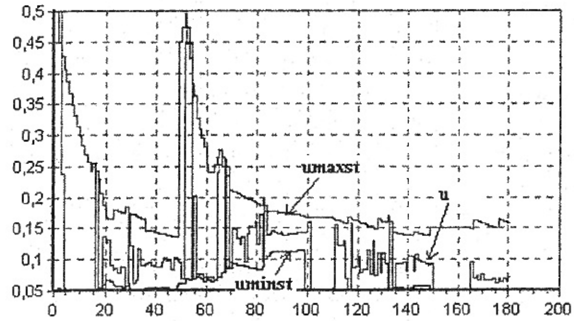

Fig. 6: Controller output (adaptive case)

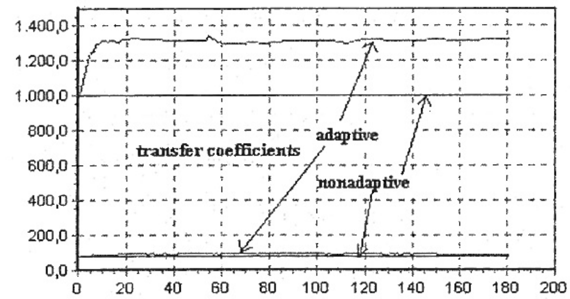

Fig. 7: Parameters identification

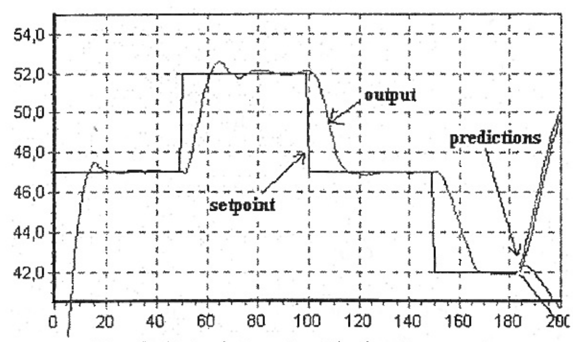

Fig. 8: Setpoint, output (adaptive case)

Initial the temperature of cold and hot fluids is $20°$. The evolution of the estimations of heat transfer coefficients is presented in figure 7. To obtain these estimations, both rotations and translations of temperature distributions and rule based correction of heat transfer coefficients are used.

In figure 8 it is used the same conditions for heat transfer coefficients, but it is not used the rotations and translations of temperature distributions. As a result, the quality of control algorithm decreases.

V. THE MODEL OF THE INVERTED PENDULUM ON A CART

The cart-pole system considered in this paper is seen in figure 9. It consists of a cart that moves on a horizontal track of finite length. The pole is represented by a point mass attached to the end of a massless thin rod of length l that is attached to the cart at a pivot capable of unconstrained (360°) rotation.

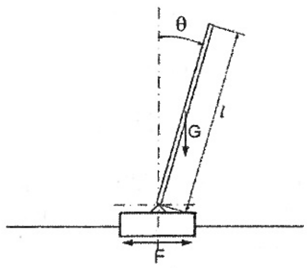

Fig.9: The inverted pendulum on a cart

The state space equation of the inverted pendulum on a cart is given by [13] :

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{(M+m)g\sin x_1 - \left(mlx_2^2 \sin x_1 - bx_4 + u\right)\cos x_1}{l\left(M + m\sin^2 x_1\right)} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{-mg\sin x_1 \cos x_1 + mlx_2^2 \sin x_1 - bx_4 + u}{\left(M + m\sin^2 x_1\right)} \end{aligned} \tag{26}$$

where $x_1 = \theta$ is the angle of the pendulum, $x_2 = \dot{\theta}$, $x_3 = x$ the position of the cart, $x_4 = \dot{x}$ and u is applied force to the cart.

The primary control objective is to stabilize the system at $[x, \dot{x}, \theta, \dot{\theta}] = [0,0,0,0]$, starting from $[0,0,\pi,0]$. This is a complicated control problem as the control is discontinuous at $\theta = \pm\pi/2$. The system is actually not controllable at this point.

The cart-pole system is a common benchmark problem in the control systems literature and is a commonly seen demonstration in many control laboratories.. Early work in linear control and stabilization of unstable systems focused on the basic stabilization problem for this system.

In this paper it is used next values of parameters [13]: M=1.378 is the mass of the cart, m=0.0551 is the mass of the pendulum, l=0.325 is the length of the pendulum, g=9.81 is the gravity, b=12.98 is the coefficient of viscous friction for motion of the cart.

In a different context, the inverted pendulum model has been used as an abstraction for many physically meaningful phenomena.

A hypothesis in the biomechanics community is that a model known as the Spring Loaded Inverted Pendulum is the control target for the musculoskeletal system. Some researchers have implemented successful walking robots based on this principle and suggest that an intuitive control algorithm designed from task specifications would be of value to many communities, such as robotics and biomechanics [15].

VI. THE CONTROL ALGORITHM APPLIED TO THE INVERTED PENDULUM ON A CART

To apply the algorithm to the inverted pendulum on a cart, first it is necessary to use a supplementary rule which approximates the sign of the process:

Rule 6: If $e_1(t_c)>e_2(t_c)$ or $e_3(t_c)>e_4(t_c)$
Then the sign is negative
Else the sign is positive (27)

where t_c is a parameter of the control algorithm. If the sign is negative, the rules (1)..(5) have a different but similar form. Second, it is used supplementary control sequences:

$$u_5(t)=\{u_{\min}, 0, .., 0\}$$
$$u_6(t)=\{0, 0, .., 0\}$$
$$u_7(t)=\{u_{\max}, 0, .., 0\} \quad (28)$$

Using the sequences (28), the rule (5) can be modified:

Rule 5.1: If $e_5(t_c)e_7(t_c)<0$
Then
$$u(t)=\frac{u_{\min st}(t)e_7(t_c)-u_{\max st}(t)e_5(t_c)}{e_7(t_c)-e_5(t_c)} \quad (29)$$
Else Rule 5

In the first experiment, the objective of the control system is only to bring the pendulum up. The figures 10..12 present the results of the control system. The sample period is 0.1 s. In fig. 10, the predictions of output are marked with the number of control sequences.

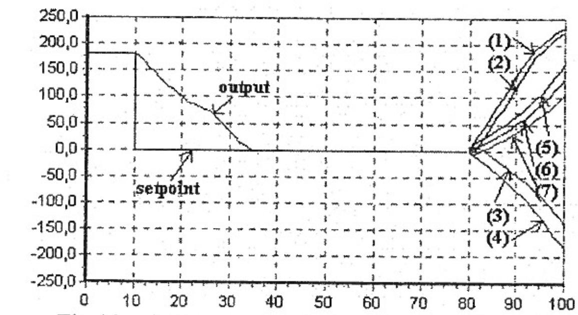

Fig.10: The setpoint, output, predictions. Case 1.

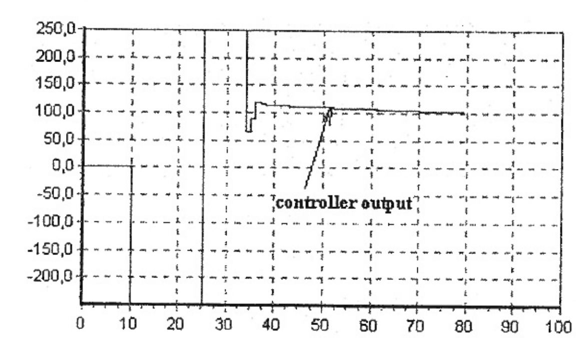

Fig. 11: The controller output. Case 1.

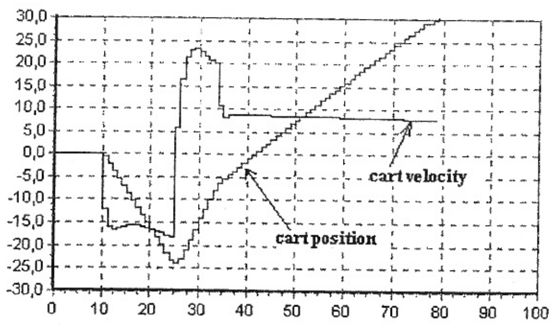

Fig. 12: The cart position and velocity. Case 1.

In the second experiment, a supplementary objective is to stop the cart ($\dot{x}=0$). A solution to obtain this is to accept the overshoot of the output $x_1=\theta$. Next rules are used in this case:

Rule 0: If $e_5(t_c)e_7(t_c)<0$ and $e(t)<\theta_p$
Then If $e_6(tc)e_5(tc)<0$
Then
$$u(t)=\frac{u_{\min st}(t)e_6(t_c)}{e_6(t_c)-e_5(t_c)} \quad (30)$$
Else
$$u(t)=\frac{u_{\max st}(t)e_6(t_c)}{e_6(t_c)-e_7(t_c)} \quad (31)$$
Else If $e(t)<\theta_q$ and $e_5(t_c)e_7(t_c)>0$
Then If $e_6(t_c)>0$ Then $u(t)=u_{\max st}(t)$
Else $u(t)=u_{\min st}(t)$
Else Rule 1, 2, 3, 4 (relations 5..8)

The figures 13..15 present the results of the control system in this case.

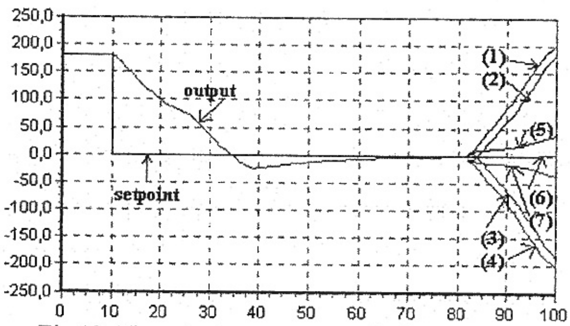

Fig.13: The setpoint, output, predictions. Case 2.

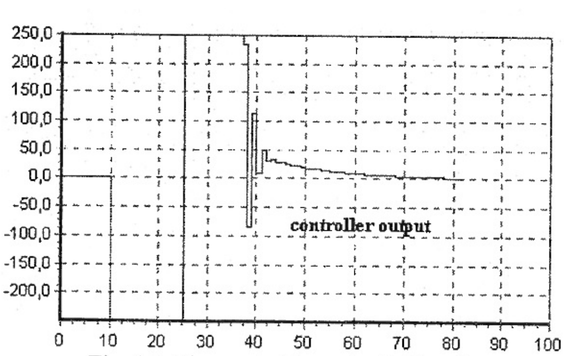

Fig. 14: The controller output. Case 2.

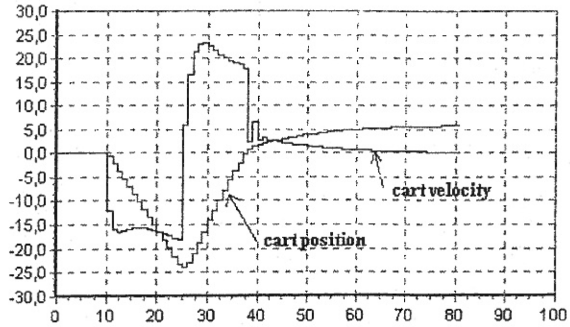

Fig. 15: The cart position and velocity. Case 2.

In the third experiment, a supplementary objective is to bring the cart in initial position (x=0). A very simple solution is to use a proportional term after relations (30), (31):

$$u(t) \leftarrow u(t)+k_p x \qquad (32)$$

The parameters t_c, θ_p, θ_q, k_p can be choosed in large limits. In the experiments it is used t_c=10, θ_p=80, θ_q=25, k_p=10.

The figures 16..18 present the results of the control system in this case.

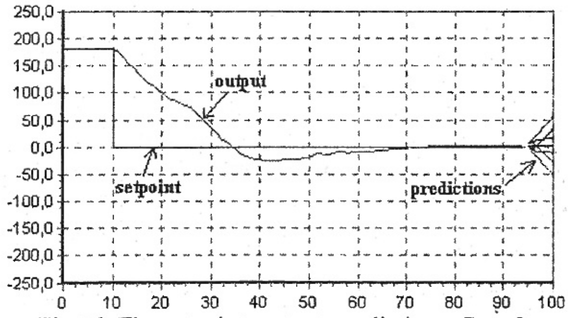

Fig.16: The setpoint, output, predictions. Case 3.

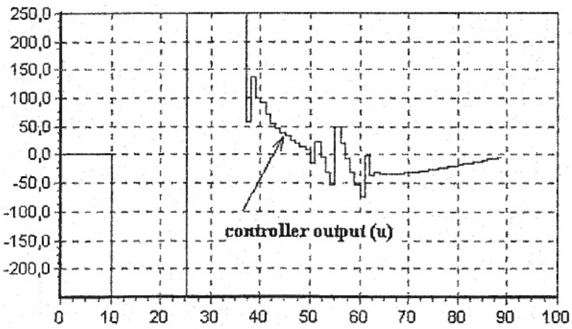

Fig. 17: The controller output. Case 3.

VII. CONCLUSIONS

A model based predictive control algorithm is presented. The algorithm uses on-line simulation and rule-based control. The application and benefits of this algorithm is demonstrated through simulation examples for two non-linear processes. The algorithm is simple to implement and requires minimal computational time.

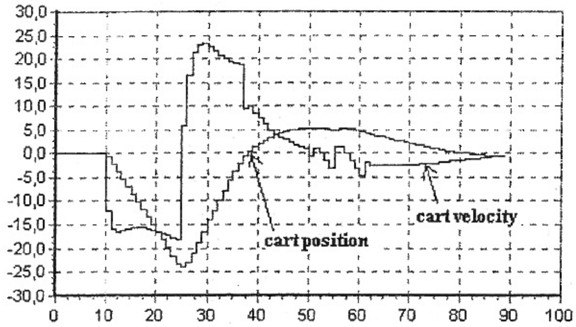

Fig. 18 The cart position and velocity. Case 3.

VIII. REFERENCES

[1] Camacho E., Bordons C. [1999], "Model Predictive Control" Spriger-Verlag

[2] Balan R. "Adaptive control systems applied to technological processes", Ph.D. Thesis 2001, Technical University of Cluj-Napoca, Romania.

[3] Dougherty, D., Cooper, D., "A practical multiple model adaptive strategy for a single loop", Control Engineering Practice 11 (2003) pp. 141-159

[4] Fischer M., Nelles O., Fink A., "Adaptive Fuzzy Model Based Control" Journal a, 39(3), Pp22-28, 1998

[5] Fink A., Topfer S., Isermann O., "Neuro and Neuro-Fuzzy Identification for Model-based Control", IFAC Workshop on Advanced Fuzzy/Neural Control, Valencia, Spain, Pages 111-116, 2001

[6] Ozisik M. N., "Heat Transfer - A Basic Approach", McGraw-Hill Book Company 1985.

[7] Douglas I.M., "Process dynamics and control", Prentice Hall Inc. 1972.

[8] Bălan, R., Mătieş, V., Hodor, V., Zamfira, I., "Some issues in the design of adaptive-predictive controllers based on on-line simulation", International Conference "OPTIM" Brasov, Romania may 2002,pp 447-452

[9] Dougherty, D., Cooper, D., "A practical multiple model adaptive strategy for a single loop", Control Engineering Practice 11 (2003) pp. 141-159

[10] Bălan, R., Mătieş, V. "Model based predictive control applied to heat exchangers", International Conference ICMA 11-13 Sept. 2002, Tampere, Finland, pp 93-100

[11] Dorf, R., Bishop, R. "Modern Control Systems" 6th edition, Addison Wesley, 1992

[12] Zilouchian, A., Jamshidi, M.: Intelligent Control Systems Using Soft Computing Methodologies CRC Press [2001]

[13] Ronco, E., Arsan T., Gawthrop P., Hill D., "A Globally Valid Continuous-Time GPC Through Successive Linearisations"

[14] Olfati-Saber R. Fixed Point Controllers and Stabilization of the Cart-pole System and the Rotating Pendulum

[15] Ramamoorthy S., Kuipers B., "Qualitative Heterogeneous Control of Higher Order Systems"

[16] Vermeiren, R., Guerra, T.M., Morère, Y., "Comparison of different fuzzy control laws of an inverted pendulum in real time"