# PINNs and Sobolev Spaces

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Abstract: Physically Informed Neural Networks (PINNs) are machine learning models that solve differential equations by embedding physical laws into the loss function of neural networks. Introduced around

2017–2018 by Raissi, Perdikaris, and Karniadakis, they unified data-driven learning with physics-based modeling. PINNs have since been applied to a wide range of ODEs and PDEs in fluid dynamics, biology, and engineering. Current research enhances their robustness, efficiency, and ability to handle complex systems and noisy data. Sobolev spaces are function spaces that generalize the concept of differentiability and integrability, allowing the study of functions with weak derivatives.

Developed in the 1930s by Sergei Sobolev, they became foundational in the theory of partial differential equations (PDEs) and functional analysis. Sobolev spaces underpin the mathematical framework of PINNs by justifying the inclusion of derivative terms in the loss function. The augmented loss penalizes deviations in function values and their weak derivatives, aligning with Sobolev norms. This promotes smoother, more physically consistent solutions. Future work may exploit this by designing Sobolev-norm-based regularization or training in Sobolev spaces. Such strategies could improve convergence, generalization, and robustness to noise in PINNs.

Short CV: Andrea De Gaetano is a biomathematician with Director of Research (full professor) tenure with CNR since 2001, currently working as Director of the CNR Institute for Biomedical Research and Innovation in Palermo. A certified emergency surgeon by training (Italy), he attained both M.Sc. (USA) and Ph.D. (France) degrees in Applied Mathematics, is a Juris Doctor with Italian Bar license and Doctor Honoris Causa (BioStatistics). He is Adjunct Professor of Mathematical Statistics, Mahidol University Dept. of Mathematics, Bangkok Thailand as well as Distinguished Professor (Full Professor with research endowment) with Obuda University, Budapest.

His research interests focus on the mathematical modelling of physiological systems with ODEs, Stochastic and Fractional differential equations, and on the attending estimation of the model parameters from experimental observations. He has published so far over 270 full-length papers on international peer-reviewed journals, largely on the mathematical modelling of energy metabolism (Google Scholar metrics: 282 documents, 12163 citations, h-index 47). He has taught Mathematical Statistics at the Universities of Urbino, Copenhagen and Mahidol Bangkok. He has obtained approx 5.1 MEuro funding for research through a series of EC-financed FP and H2020 projects, foreign financed grants and several Italian projects (Ministry of Research, Ministry of Defense). He is Past President of the European Society for Mathematical and Theoretical Biology, and has served as the Italian National Academic delegate to the NATO Scientific and Technical Organization, Human Factors and Medicine Panel and to EDA European Defense Agency, Captech Simulation.